European Journal of Educational Research
Volume 7, Issue 2, 373 - 385.
ISSN: 2165-8714
http://www.eu-jer.com/

Fraction Multiplication and Division Word Problems Posed by Different Years of Pre-Service Elementary Mathematics Teachers

Tuba Aydogdu Iskenderoglu *
Karadeniz Technical University, TURKEY

Received: January 31, 2018 ▪ Revised: March 8, 2018 ▪ Accepted: March 30, 2018

Abstract: It is important for pre-service teachers to know the conceptual difficulties they have experienced regarding the concepts of multiplication and division in fractions and problem posing is a way to learn these conceptual difficulties. Problem posing is a synthetic activity that fundamentally has multiple answers. The purpose of this study is to analyze the multiplication and division of fractions problems posed by pre-service elementary mathematics teachers and to investigate how the problems posed change according to the year of study the pre-service teachers are in. The study employed developmental research methods. A total of 213 pre-service teachers enrolled in different years of the Elementary Mathematics Teaching program at a state university in Turkey took part in the study. The "Problem Posing Test" was used as the data collecting tool. In this test, there are 3 multiplication and 3 division operations. The data were analyzed using qualitative descriptive analysis. The findings suggest that, regardless of the year, pre-service teachers had more conceptual difficulties in problem posing about the division of fractions than in problem posing about the multiplication of fractions.

Keywords: *Pre-service mathematics teachers, fractions, problem posing, multiplication, division.*

To cite this article: Aydogdu Iskenderoglu, T. (2018). Fraction multiplication and division word problems posed by different years of pre-service elementary Mathematics teachers. *European Journal of Educational Research*, *7*(2), 373-385. doi: 10.12973/eu-jer.7.2.373

Introduction

Teaching involves everything that teachers do to support their students, including the interactive work of teaching lessons in classrooms and all the tasks that arise in the course of that work. At the same time, teaching also involves planning for those lessons, making and managing homework, evaluating students' work, attending to concerns about equality, explaining the classwork to parents, writing and grading assessments, and dealing with the school principal, who often has strong views about the mathematics curriculum (Ball, Thames & Phelps, 2008). To assume all these tasks, it is important that teachers be qualified. The studies reviewing the qualifications of teachers are often designed on the basis of the work done by Shulman (1987), who noted that the various types of field knowledge a teacher is expected to have involve references to basic concepts and principles that field, the curriculum, and the relationships between these. Furthermore, the teachers' behavior in the classroom and their practices during the activities are also shaped by their content knowledge (Ball & Bass, 2000). That is why content knowledge plays such a major role in shaping the process of education in the classroom (Ball, Lubienski & Mewborn, 2001; Cakan, 2004). The knowledge to be provided and the means to provide that knowledge are important, for the effectiveness of the teacher is among the major factors affecting the students' learning (Romberg & Carpenter, 1986). In mathematics some concepts are difficult to learn for both teachers and students. The concept of rational numbers is one case in point that presents particular challenges to teach as well as to learn.

The rational numbers set includes, in addition to natural numbers, fractions and decimals. The fractions, in turn, constitute a fundamental element of decimals, rational numbers, ratio and proportion, and measurement systems. Fractions, which serve a key function in the field of mathematics, are complicated as a cognitive object and are among the more difficult mathematical concepts to understand for students, pre-service teachers and to teach for teachers (Isik, Isik & Kar, 2011; Ma, 1999 Tirosh, 2000; Yim, 2010).

Learning to pose mathematical tasks is one of the challenges of learning to teach mathematics (Crespo, 2003). The studies on problem posing indicate that pre-service teachers' level of success in demonstrating problem posing skills is generally low (Isik, 2011; Isik, Isik & Kar, 2011; Toluk-Ucar, 2009). For professional development of pre-service teachers' we have to develop their mathematical knowledge structures during their education, and it is possible to achieve this goal through teacher

* **Correspondence:**
Tuba Aydogdu Iskenderoglu , Karadeniz Technical University, Fatih Faculty of Education, Trabzon, Turkey.
Email: tiskenderoglu@ktu.edu.tr

education programs (Kilic, 2015). In Turkey, pre-service elementary mathematics teachers learn how to teach mathematics in elementary mathematics teacher education programs through method courses such as Mathematics Methods I and Mathematics Methods II. One of the topics they will subsequently teach in elementary mathematics is problem posing including many mathematical concept such as fractions. However, one of the fundamental aims of mathematics courses in elementary education is to instill problem solving skills, and to achieve this, teachers must pose problems. Instilling mathematics problem posing skills provide many benefits to teachers and also pre-service teachers. For instance, in cases where the problems offered in textbooks are insufficient or inappropriate for the skills of the students, or do not reflect the interests and needs of the students, the teacher would be required to pose original problems regarding the topic in order to further their teaching (Korkmaz & Gur, 2006). Additionally, the problem posing skills also have an impact on the process of shaping proper behavior in the conception stage. Pre-service teachers are the teachers of the future. If the pre-service teachers are equipped with such problem posing skills, they would be able to make good use of these skills in their teaching work. Therefore, it is necessary to raise teachers who are not only aware of the importance of the problem posing perspective but who also endowed with the fundamental notions and skills (Akay, Soybas & Argun, 2006).

Moreover, the results of the study done by Isik (2011) on multiplication and division with fractions indicate that pre-service teachers have difficulty with the concept of fractions and with the concept of operations of fractions. These conceptual difficulties have been observed in pre-service elementary mathematics teachers who are fourth-year students and soon expected to lead classroom teaching activities. In this sense, it is important for pre-service teachers to be aware of the conceptual difficulties they have with the overall concept of fractions as well as the concepts of multiplication and division in fractions (Isik, 2011). This is why it is crucial to conduct analyses of pre-service elementary mathematics teachers at various years of their training, to determine their skills in posing problems regarding multiplication and division with fractions, and to identify any shortcomings. Pre-service mathematics teachers' skills in posing problems regarding multiplication and division with fraction is chosen for two reasons. First, previous researches suggest that teachers' and as well as pre-service teachers' understanding of fractions with division and multiplication is limited (Ball, 1990; Tirosh, 2000; Toluk-Ucar, 2009). But there aren't enough studies how these limitations change through different years of teacher training programme. Second, as Toluk-Ucar (2009) noted that fraction is semantically rich for teaching and learning because several conceptual meanings exist within the domain. If pre-service teachers can able to pose appropriate problems with fractions, then we can say that they know the conceptual meaning of fractions. Accordingly, the purpose of the present study is to provide a conceptual analysis of the problems posed, with respect to multiplication and division with fractions, by pre-service elementary mathematics teachers who are in different years of a teacher training program, investigating the variation of such problems with reference to the year of training. We are trying to understand, analyze and interpret the results of the analysis similarities or differences between pre-service teachers who are in different years. In addition to this, a developmental research method is also investigating questions such as what and what happened. The results of this study can provide support for the conceptual meaning of fractions necessary for quality mathematics teaching and teacher education.

Problem Posing

Problem solving and posing skills are among the most fundamental skills in any mathematics curriculum in Turkey (Ministry of National Education (MEB, 2005; MEB, 2013; MEB, 2017), as well as throughout the world (Abramovich, 2014; Chen, Dooren, Chen & Verschaffel, 2011). The primary and secondary school mathematics curriculums developed in Turkey after 2004 have focused on mathematics teaching-learning activities based on the problem solving-posing perspective (MEB, 2005; MEB, 2013; MEB, 2017). The education programs currently in place aim to raise individuals who pose questions, investigate matters, are capable of critical thought and asking questions, as well as, solving problems faced in daily life.

Problem solving is a process, in which a learner reaches a correct answer from a mathematical structure from given information. In contrast, problem posing is a synthetic activity that fundamentally has multiple answers, and as such, it requires creative thinking between multiple answers (Kojima, Miwa & Matsui, 2009).

Problem posing is an important mathematical activity that has several benefits (Cankoy & Darbaz, 2010; Kilic, 2013a; Toluk-Ucar, 2009; Turhan & Guven, 2014). Through it, students are better able to understand important elements of the problem solving process, including the way mathematical ideas are related within the context. Moreover, students are provided the opportunity to reflect the mathematical understanding, skills and beliefs they have when posing problems. In this way, teachers can gain insights about students' understanding of mathematical concepts (Stoyanova, 2003; Whitin, 2004). Problem posing has long been viewed as a characteristic of creative activity or exceptional talent (Silver, 1994). Individuals who are equipped with problem posing skills develop new knowledge using existing sets of knowledge and can come up with problems of their own (Turhan & Guven, 2014). This lends to the inference that problem posing is a re-formulation of the problem and entails a search for patterns. Moreover, problem posing enables teaching of mathematical reasoning, and instills the abilities to discover mathematical cases and to formally describe mathematical cases in speech and text.

Problem posing skills can be developed, however, only when the teachers design appropriate learning-teaching processes (Akay, Soybas & Argun, 2006). For, taking into account the fact that problems posed by teachers will serve as models for the students, the variety of the problems posed will serve to enrich the teaching activities, in terms of instilling the skills required in this context (Isik, Isik & Kar, 2011).

Fractions and Posing Problem about Fractions

The subject of fractions lies at the core of many topics in mathematics. Studies reveal that rational numbers, in other words, expressions in the form "a/b", can be interpreted in more than one way (Behr et al., 1983; Behr et al., 1992; Kieren, 1976). For instance, a rational number can represent, in accordance with the problem environment, part-whole relationships, as well as measurements, divisions, and ratios. It may also serve as an operand (Toluk, 2002). At the same time there are four operations with fractions which named addition, subtraction, multiplication, and division. Students should be able to present solutions for computation with fractions in the four of these operations (Holmes, 1995).

Problem posing plays a pivotal role in establishing links between real life cases and fractions as well as operations with fractions (Abu-Elwan, 2002). Problem posing pushed people to think about the meaning of all four operations with fractions (Xie & Masingila, 2017). However, studies suggest that when asked to pose a problem involving divisions with fractions, teachers and pre-service teachers usually pose multiplication problems, or fail to correctly pose one at all (Tirosh, 2000; Toluk-Ucar, 2009; Utley & Redmond, 2008). In one study, pre-service teachers were observed to be capable of successfully calculating the expression $1\frac{3}{4} \div \frac{1}{2}$, but most of them could not pose a word problem describing this expression (Ball, 1990). All these studies generally indicate that the teachers and pre-service teachers usually have limited capabilities in terms of posing problems involving divisions with fractions that are related to real life cases (Ball, 1990; Toluk-Ucar, 2009). A few studies touch briefly on the common errors regarding multiplication with fractions (Ball, 1990; Isik, 2011; Toluk-Ucar, 2009). Toluk-Ucar (2009), in particular, noted that when asked to pose a word problem to reflect the operation $\frac{3}{4} \times \frac{1}{3}$, pre-service primary teachers usually posed a problem reflecting the operation $\frac{3}{4} \div 3$. Ball (1990), on the other hand, asked pre-service teachers to identify the problems describing division with fractions from among a larger set of word problems. Thirty percent of the pre-service teachers identified the same word problems as reflecting both the following operations: $4\frac{1}{2} \div 2$ and $4\frac{1}{2} \div \frac{1}{2}$ (Ball, 1990). Studies results show that pre-service teachers are having problems with conceptual meaning of fractions (Ball, 1990; Toluk-Ucar, 2009).

Isik (2011), in contrast to the above, focused on the conceptual analysis of the problems posed by pre-service elementary mathematics teachers, with respect to division and multiplication with fractions. Isik (2011) developed a “Problem Posing Test” which constituting of four division and four multiplication questions. Isik indicated that in the problems posed to describe operations with two improper fractions, two proper fractions, and a natural number divided and multiplied by a proper fraction. In each question of the test, Isik required from pre-service teachers to pose a problem by using the given operation and problem which was posed by pre-service teachers have to be appropriate to elementary school students’ level. In Isik’s study data were gathered through a detailed analysis of answers of each item. In this study Isik’s sub-categories used when data were analyzing. Results of Isik’s study indicate that pre-service teachers usually omitted the measurement purpose of the division, which suggested that the conceptual structure of division with fractions was not established properly in the problems. At the same time, the level of success in the posing of problems regarding division with fractions was lower compared to the level of success achieved in the solution of the problems requiring division. Moreover, Isik found that pre-service elementary mathematics teachers were usually successful in attaching meanings to operations and numbers in the problems posed for multiplication with fractions. During multiplication of a natural number and a fraction problem posing process, pre-service elementary mathematics teachers could not transfer part-whole relations to the problems posed. It was seen that conceptual meaning and the quantification property of fractions were ignored. Against this background, Isik noted that pre-service teachers had more conceptual difficulties in problem posing about division of fractions than in problem posing about multiplication of fractions (Isik, 2011). At the same time Xie & Masingila (2017) noted that among all four operations, division was the most difficult one for posing story problems. This result indicate that pre-service teachers generally had difficulties in giving meanings to operation and numbers. If these difficulties of pre-service teachers can not be solved, then their students will have similar problems when they are teachers.

Teachers and pre-service teachers have difficulties regarding the concept of fractions and at the same time division and multiplication operations with fractions (Toluk-Ucar, 2009). These difficulties also affect the problem posing process. If we can educate pre-service teachers, they can pose problems on every topic—including fractions—effectively, they will teach students how to problem solve and pose effectively. So that if we want students to be good problem posers with fractions, we have to prepare pre-service teachers well about problem posing (Ervin, 2017; Kilic, 2013b).

Fractions and Problem Posing in Turkish Mathematics Curriculum

In 2004, the school mathematics curriculum for grades 1 through 8 underwent an important change, and in 2013 and 2017 some topics of the new curriculum were updated. The new curriculum emphasis on providing students a conceptual understanding of mathematics. More particularly, the curriculum demands that teachers provide students with opportunities to mathematical reasoning, solve and pose problems, communicate, make connections among mathematical ideas, and represent mathematical ideas

When a pre-service elementary mathematics teacher becomes a teacher in Turkey, they will teach mathematics from fifth grades to eighth grades and they will have to follow a national mathematics curriculum. This mathematics curriculum has contained problem posing applications, especially with regard to the curriculum for first through fifth grades since 2006 (MEB, 2006). In the mathematics curriculum, some of the learning outcomes are related to posing fraction problems. The Turkish

mathematics curriculum emphasize that students should understand fraction concepts and the meaning of fraction operations (MEB, 2017). In the first three grades of the school, students learn comparing and ordering fractions. When students in the fourth and fifth grades, they begin to learn addition, subtraction, and multiplication operations with fractions (Kilic, 2015). Students start to experience posing problems with natural numbers from first grade. When students in the sixth, seventh and eighth grades, they begin to solve problems which include operations of fractions (MEB, 2017). For this reason, it is important that teachers have the ability to pose problems.

Research Question

Isik (2011) focused on the conceptual analysis of the problems posed by fourth year of pre-service elementary mathematics teachers, with respect to division and multiplication with fractions. Building on Isik's (2011) study, our study focused on exploring to provide a conceptual analysis of the problems posed, with respect to multiplication and division with fractions, by pre-service elementary mathematics teachers who are in different years of a teacher training program, investigating the variation of such problems with reference to the year of training. Analyzing pre-service teachers' knowledge of fractions by problem posing will help teacher educators to develop their knowledge about fractions. Doing that will help pre-service teachers to be equipped problem posing (Kilic, 2015).

Methodology

This study was designed as a cross-sectional study applying developmental research methods. Cross-sectional studies are usually characterized by their aim to compare, define, categorize, and analyze individuals, groups, organizations, methods, or materials, in order to gain insight into their differences and to interpret the results of the analysis (Menard, 2008; Miller, 1998). A cross- sectional study tests different people at different ages (Miller, 1998). This study aims to reveal the changes in the problems posed by pre-service elementary mathematics teachers at different years of their education.

Study Group

The study group was comprised of 213 pre-service elementary mathematics teachers who were at different years of teacher training in the Department of Elementary Mathematics Teaching, at a Faculty of Education of a state university in Eastern Turkey. The elementary mathematics teacher training program is four years (8 semesters) in Turkey. In terms of the distribution of the pre-service teachers according to their year in school, 47 were in their first year, 53 in their second, 57 in their third, and 56 in their fourth.

In Turkey, elementary mathematics teachers are trained in 4-year programs offered by the education faculties of universities. As part of the program, pre-service teachers are required to take mathematics content courses (Abstract Mathematics I, II; Analysis I, II, III; Linear Algebra I, II; Analytic Geometry I, II; Elementary Number Theory, Differential Equations, Algebra) and pedagogical content knowledge courses (Instructional Technologies and Materials Design, Mathematics Methods I, II). Moreover, pre-service teachers are required to take a one-year mathematics methods course in the third year of the program and this courses are usually designed to focus on how to teach elementary school mathematics.

Data Collection Tools

The data collection tool used in the study was the "Problem Posing Test". This test features a total of 6 operations, 3 multiplication and 3 division, as was the case with Isik's study (2011). The operations included in the "Problem Posing Test" are shown in Table 1.

Table 1. Problem posing test

Operations	Operations involve
$\frac{3}{4}x8$	The multiplication operation involves the multiplication of a proper fraction with an integer,
$\frac{3}{4}x\frac{5}{7}$	The multiplication operation involves the multiplication of two proper fractions,
$1\frac{1}{3}x\frac{1}{4}$	The multiplication operation involves the multiplication of a mixed number with a proper fraction equal to a quarter,
$5 \div \frac{5}{8}$	The division operation involves the division of an integer by a proper fraction,
$\frac{11}{12} \div \frac{1}{4}$	The division operation involves the division of a proper fraction by another proper fraction equal to a quarter,
$3\frac{4}{5} \div 1\frac{1}{10}$	The division operation involves the division of a mixed number by another mixed number.

There are 3 multiplication operations and 3 division operations in the "Problem Posing Test" (see Table 1.). All of the operations involve different fractions so we can see what kinds of problems that pre-service teachers pose in different fractions. The "Problem Posing Test" was presented to each pre-service teacher in written form. Under each mathematical operation, there was space available to allow the pre-service teachers to take notes and make calculations.

Data Collection

Once the test was given to the pre-service teachers, they were asked to pose a word problem that required the use of the specified operation. For each question of the test, pre-service teachers were required to write a word problem representing the given operation. The subject matter of the problem was not important, that is, pre-service teachers could lay out a context or write a story problem for the operations. The respondents were told that they were required to pose problems appropriate for the elementary school students' level, and it was recommended that they leave the question unanswered, if they felt unable to pose a proper problem. The "Problem Posing Test" was applied in the form of an exam, taking an hour's worth of class time to complete, in order to prevent the respondents from affecting the responses of others.

Data Analysis

The problems posed by the elementary pre-service teachers were analyzed using the qualitative descriptive analysis method, with reference to the problem categories identified by Isik (2011). Each problem sentence posed by the pre-service teachers was read and reviewed carefully, and classification of the sentences was carried out by coding them. As part of the review process, the problems were analyzed by the author and two Ph.D. students. The analyses performed by the author and the researchers were changed from 85% to %93 consistency rate in the comparisons made between their analyses. The items on which all three reviewers concurred were assumed to be analyzed accurately, whereas the ones subject to a divergence of assessment were discussed in depth in order to come up with a unanimous assessment. In the classification of the sentences, categories were formed from problems which were thought to have the same meaning, and each problem was included in only one category. These categories are listed in the "Findings" section. The problems that defied solutions by simply noting down the mere application of the specified operations and the problems that had missing elements or were unable to be understood, were marked as "Incorrect statement". The others were placed in the problem category, but while some of them gave the result of the given operation, others failed to give the result of the given operation. The categories established through the analysis and the relevant frequency tables are shown in the Findings section.

Findings

In this section, first the findings regarding the problems posed for multiplication with fractions that were presented by elementary pre-service mathematics teachers who were at different years of their training are detailed below, followed by the findings regarding the problems posed for division with fractions.

Findings Regarding the Problems Posed for Multiplication

In this section, the categories of problems posed by the pre-service teachers for multiplication will be presented in separate tables for each expression. These tables will show the distribution of the problems posed according to the students' year in school. The problems related to the multiplication of a proper fraction and a natural number that were posed by the pre-service elementary mathematics teachers (arranged according to their year of education) were found to vary (see Table 2.).

Table 2. The categories for the problems posed by pre-service mathematics teachers for the operation "$\frac{3}{7} \times 8$", according to the teachers' year in school

Categories \ Years	First year (n=47)	Second year (n=53)	Third year (n=57)	Fourth year (n=56)	Total (n=213)
	%	%	%	%	%
Repeated addition	17	51	56	20	37
Imagining a unit in 8 equal parts	40	28	11	20	24
Simple exercises	15	6	7	38	17
Proportion	2	4	2	9	4
Incorrect statements	13	11	21	12	14
Empty	13	-	4	-	4
Total	100	100	100	100	100

Table 2 reveals that among the problems posed by the pre-service teachers, most fell under the category of *Repeated addition* (37%). The following serves as an example of this type of problem: "Each child will be given $\frac{3}{7}$ of a cake. There are 8 children. How many cakes are needed?". Repeated additions were used most frequently by third (56%) and second year (51%) students, whereas the first (17%) and fourth year (20%) students used them relatively rarely. The second most frequently posed problems (24%) fell under the category *Imagining a unit in 8 equal parts*, where the problem involved trying to determine the extent corresponding to $\frac{3}{7}$ of the whole. Such problems were usually posed along the lines of "How long is $\frac{3}{7}$ of an 8 meter-long rope?". First year (40%) students constituted the largest number of participants to pose problems in this category, followed by second (28%), fourth (20%), and third year (11%) students. Some (17%) of the pre-service teachers posed problems which

were essentially simple exercises, such as "Please calculate the result of the multiplication of 8 by $\frac{3}{7}$.". Problems posed as simple exercises were most commonly seen in fourth year (38%) students, followed by first (15%), third (7%), and second year (6%) students. A number of respondents (14%), on the other hand, posed incorrect problems, which was observed in 21% of third, 13% of first, 12% of fourth and 11% of second year students. In this context, the problems posed by these students could not be solved through the application of the required mathematical expression, whereas in some cases, certain data was missing, or operations which were not required appeared to be needed in the solution. Furthermore, in some problems, the pre-service teachers failed to understand the whole as the multitude of units. For instance, in the problem "Ahmet gave 8 times $\frac{3}{7}$ of the 70 beads he has to his brother. How many beads did he give to his brother?", even though Ahmet had 70 beads to begin with, he is supposed to give 240 of those to his brother.

The categories under which the problem posing was arranged varied by the year of training the pre-service teacher received. While the first year students had a decisive preference for the category of problems envisaging the whole in 8 units, and trying to calculate $\frac{3}{7}$ of it, half of the second year students and more than half of the third year students chose to use repeated additions. The fourth year students, in contrast, had a preference for simple exercises to pose problems. Incorrect statements were most frequently made by the third year students, while the most unanswered questions were seen in the first year students. The problems posed by the pre-service elementary mathematics teachers at different years of training, with respect to the multiplication of two proper fractions, are summarized in Table 3.

Table 3. The categories for the problems posed by pre-service mathematics teachers for the operation "$\frac{3}{4}x\frac{5}{7}$", according to the teachers' year in school

Categories \ Years	First year (n=47)	Second year (n=53)	Third year (n=57)	Fourth year (n=56)	Total (n=213)
	%	%	%	%	%
Calculation of $\frac{5}{7}$ of $\frac{3}{4}$ of a plurality	11	19	28	5	16
Calculation of $\frac{5}{7}$ of three fourths of a whole	19	21	35	34	28
Simple exercises	21	33	11	36	25
Proportion	4	2	9	4	5
Through division	-	6	-	2	1
Incorrect statements	9	15	15	19	15
Empty	36	4	2	-	10
Total	100	100	100	100	100

Table 3 reveals that the largest group (28%) of pre-service teachers posed problems requiring the calculation of $\frac{5}{7}$ of $\frac{3}{4}$ of a whole. An example of this category of problems is as follows: "Serkan plows $\frac{3}{4}$ of his field. Then he sows maize in $\frac{5}{7}$ of the plowed area. What is the proportion of the area of maize cultivation to the whole field?". Third year (35%) students were the ones who most frequently posed this type of problem, while 34% of fourth, 21% of second, and 19% of first year students did so as well. A fourth (25%) of the participants posed simple exercises, which were usually in the form of "What is $\frac{3}{4}$ of $\frac{5}{7}$?". Thirty-six percent of fourth year students posed simple exercises, followed by 33% of second, 21% of first, and 11% of third year students. Sixteen percent of the pre-service teachers posed problems requiring the calculation of $\frac{5}{7}$ of $\frac{3}{4}$ of a plurality. The problems posed by third year (28%) students fell under this category most frequently, followed by 19% of second, 11% of first, and 5% of fourth year students. The problems posed by 15% of the participants included incorrect statements. As an example, one pre-service teacher posed the following problem: "A cow produces 4 liters of milk per day. Ayse sells 3 liters out of the 4. Then, she divides the remaining liter into 7 bottles, and sells 5 of such bottles. How much milk was sold in total?". This problem does not reflect the specified expression. A different set of operations is required to solve it. Problems containing incorrect statements were posed most frequently by fourth year students (19%), followed by 15% of both second and third year students, and 9% of first year students. Moreover, 10% of the participants did not write problems to the question, a failing most seen in the first year students (36%).

An analysis of the categories formed for the problems posed by the pre-service teachers at different years of education revealed that the third year students had the most problems posed under the *Calculation of $\frac{5}{7}$ of $\frac{3}{4}$ of the whole or a plurality* category. Fourth year students were found to most prefer simple exercises, and out of all the pre-service teachers, they had the most incorrect statements, while the lowest rate of incorrect statements was seen in the first year students. Second and third year students, on the other hand, almost had as many incorrect statements as the fourth year students, and first year students were the group to most frequently leave it blank. Very few pre-service teachers in the second or third year left the question empty. The problems posed by pre-service elementary mathematics teachers according to their different years of training, with respect to the multiplication of a mixed number with a proper fraction, are summarized in Table 4.

Table 4. The categories for the problems posed by pre-service mathematics teachers for the operation "$1\frac{1}{3}x\frac{1}{4}$", according to the teachers' year in school

Categories \ Years	First year (n=47)	Second year (n=53)	Third year (n=57)	Fourth year (n=56)	Total (n=213)
	%	%	%	%	%
Calculation of $\frac{1}{4}$/one fourth of $1\frac{1}{3}$	12	20	39	22	24
Calculation of $\frac{1}{4}$/one fourth of $\frac{4}{3}$	4	4	4	7	5
Through division	4	26	14	20	16
Simple exercises	6	21	15	36	20
Help of a diagram	3	-	-	-	1
Incorrect statements	16	21	14	13	16
Empty	55	8	14	2	18
Total	100	100	100	100	100

The problems posed by 24% of the participants for the calculation of $\frac{1}{4}$ one fourth of $1\frac{1}{3}$ (see Table 4.) were similar to the following: "Hasan was very hungry and ate 1 cake and also one third of another cake. A friend of his, in turn, ate one fourth of the cake Hasan ate. What is the proportion of the amount of cake the friend ate, compared to Hasan's?". Thirty-nine percent of the pre-service teachers in their third year of education posed problems that fell under this category, followed by 22% of fourth, 20% of second, and 12% of first year students. One-fifth (20%) of all the pre-service teachers posed simple exercises for the multiplication of a mixed number with a proper fraction. Simple exercises were employed mostly by fourth year (36%) students, while only 21% of second, 15% of third and 6% of first year students did so. Such exercises were usually in the form of "What is $\frac{1}{4}$ times 1 and $\frac{1}{3}$?". Eighteen percent of the pre-service teachers left this question unanswered, with first year students making up a large percentage (55%) of those who did not answer the question. Fourteen percent of third, 8% of second, and only a few (2%) of the fourth year students provided no response. Sixteen percent, on the other hand, posed problems which could be solved through division help. These problems were often variations of the following example: "There is one full cake, and also one third of a cake. Four persons will share these. How much cake would each get?". The problems posed with reference to division were most frequently produced by second year (26%) students and least frequently produced by first year (4%) students. Sixteen percent of the participants posed problems containing incorrect statements. For instance: "Aylin divides 4 loaves of bread into 3 equal pieces, each to be placed on a table. As 4 guests are seated at each table, how much bread would each guest get?". The posing is incomplete, as the number of tables is not clear; hence the solution is not possible. Twenty-one percent of second, 16% of first, 14% of third and 13% of fourth year students posed problems containing incorrect statements.

The problems posed by pre-service teachers with respect to the multiplication of an integer and a proper fraction also varied in terms of the categories they fell under. A majority of the third year students' problems fell under the category *calculation of* $\frac{1}{4}$*/ one fourth of* $1\frac{1}{3}$, whereas division- based problems were most common with the second year students. Fourth year students had a marked preference for simple exercises. Even though more than half of the first year students left the question unanswered, much lower numbers did so among the second and third year students. The number of fourth year students who left the questions unanswered was very low. Incorrect problem posing was seen most often with the second year students and least often with the fourth year students.

Findings Regarding the Problems Posed for Division with Fractions

In this section of the study, the categories formed for the problems posed by the pre-service teachers for division will be presented in separate tables for each operation. These tables will show the distribution of the problems posed according to the students' year in school. The problems posed for the division of an integer by a proper fraction by pre-service elementary mathematics teachers at different years of their education are summarized in Table 5.

Table 5. The categories for the problems posed by pre-service mathematics teachers for the operation "$5 \div \frac{5}{8}$", according to the teachers' year in school

Categories \ Years	First year (n=47)	Second year (n=53)	Third year (n=57)	Fourth year (n=56)	Total (n=213)
	%	%	%	%	%
Measurement	-	36	23	14	19
Proportion	-	11	8	-	5
Simple exercises	6	7	11	20	11
Through multiplication	9	6	5	18	9
Confused multiplication-based operations	9	6	17	32	16
Incorrect statements	4	23	19	11	15
Empty	72	11	17	5	25

Total	100	100	100	100	100

Almost $\frac{1}{4}$ (25%) of all pre-service teachers who took part in the study left the question unanswered without formulating a problem (see Table 5.). No-response was most frequent with the first year (72%) students, in contrast to the fourth year (5%) students, who had the least number of no-responses. On the other hand, 19% focused on the measurement side of the operation, applying consecutive subtraction. For instance, the solution to the problem "A 5 cm long bar is to be divided into 5/8 cm long parts. How many parts do we get?" involves finding $\frac{5}{8}$ cm parts within the bar. Thirty- six percent of second year students posed problems that fell under the measurement category, followed by 23% of third and 14% of fourth year students. Sixteen percent of the pre-service teachers confused the problems with multiplication-based operations. Confusion with multiplication-based problems was most prevalent with fourth year students (32%), followed by 17% of third year students. The problems posed by these pre-service teachers included ones such as "Ayse will give five-eighths of her 5 pencils to her sister. How many pencils will she give to her sister?" The solution to this problem requires the operation $5 \times \frac{5}{8}$, although the required operation was $5 \div \frac{5}{8}$. The problems posed by 15% of the pre-service teachers contained incorrect statements. Twenty-three percent of second and 19% of third year students provided incorrect statements. Fewer students in other years of education posed such problems. Simple exercises were posed by 11% of the pre-service teachers, with statements such as, "Please calculate the result of the division of 5 by $\frac{5}{8}$.". These kinds of problems were posed mostly by fourth year students (20%), while the problems posed by first year students (6%) did not fall under this category very much. The problems formed *through multiplication* (9%) were variations of the following example: "What is one-fifth of 8 times 5?", and these types of problems were again employed mostly by fourth year students (18%).

The categories for the problems posed by the pre-service elementary mathematics teachers on the topic of division of an integer by a proper fraction varied by the years in the program. A vast majority (72%) of first year students failed to pose a word problem to reflect the operation. However, among the fourth year students, this figure was much lower (5%). Second and third year students frequently employed measurement problems, while the fourth year students' problems tended to fall under the category of posing problems through multiplication. The second year students were most frequently seen to pose incorrect problems, while the first year students were the least. Simple exercises were most frequently employed by fourth year students, while first year students did not ever employ them. The problems posed on the division of a proper fraction by another proper fraction equal to one quarter by pre-service elementary mathematics teachers are summarized in Table 6.

Table 6. The categories for the problems posed by pre-service mathematics teachers for the operation "$\frac{11}{12} \div \frac{1}{4}$", according to the teachers' year in school

Categories \ Years	First year (n=47)	Second year (n=53)	Third year (n=57)	Fourth year (n=56)	Total (n=213)
	%	%	%	%	%
Measurement	-	30	12	9	13
Proportion	2	10	9	9	8
Through multiplication	6	6	7	17	9
Confused multiplication-based operations	11	7	9	27	14
Simple exercises	4	19	22	29	18
Incorrect statements	6	9	7	9	8
Empty	71	19	37	-	30
Total	100	100	100	100	100

Thirty percent of the pre-service teachers left this question unanswered, failing to pose a corresponding problem (see Table 6). Broken down according to year in school, 71% of the first year students left this question unanswered, while none of the fourth year students did so. Eighteen percent of the pre-service teachers posed problems in simple exercise form and employed statements such as "Please state the fraction to be obtained by dividing $\frac{11}{12}$ by $\frac{1}{4}$". Fourth year students (29%) used these problems most frequently, in contrast to first year students, among whom just 4% posed problems that fell under this category. Fourteen percent of the respondents confused the operation with a multiplication one, as in the example of "What is $\frac{1}{4}$ of $\frac{11}{12}$?". Thirteen percent of the respondents posed problems for the *Measurement* category. Measurement-based problems, such as "Melek ate $\frac{11}{12}$ of a cake. If you wanted to distribute the cake Melek ate to her friends at a rate of $\frac{1}{4}$; how many friends would you need?", were posed mostly by second year students (30%), while first year students did not pose any problems that fell under this category. Moreover, 9% of the pre-service teachers posed problems through multiplication, such as "What is 4 times $\frac{11}{12}$?", while 8% of the participants posed problems that fell under the proportion application category and 8% posed problems with incorrect statements.

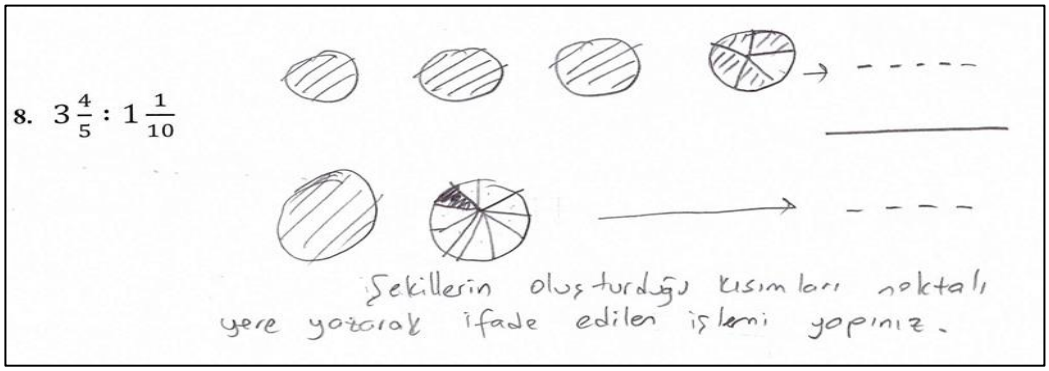
Out of all the pre-service teachers, the ones who left this question unanswered most frequently were the first year students, while none of the fourth year students did so. The unanswered question rate of the third year students was higher than that of the second year students. Second year students had the most problems posed under the *Measurement* category, followed by the

third and fourth year students; first year students did not pose any problems that fell under the *Measurement* category. The simple exercises were most frequently used by the fourth year students. The rate of simple exercises fell in parallel to the years in the program. Confusion with multiplication was most prevalent with the fourth year students, but relatively rare in other years. The problems regarding the division of a mixed number by another mixed number, posed by pre-service elementary mathematics teachers, are summarized in Table 7.

Table 7. The categories for the problems posed by pre-service mathematics teachers for the operation "$3\frac{4}{5} \div 1\frac{1}{10}$", according to the teachers' year in school

Categories \ Years	First year (n=47)	Second year (n=53)	Third year (n=57)	Fourth year (n=56)	Total (n=213)
	%	%	%	%	%
Measurement	-	11	14	9	9
Proportion	2	19	12	9	11
Simple exercises	2	17	11	38	17
Through multiplication	-	2	2	9	3
Confused multiplication-based operations	4	4	7	23	10
Help of a diagram	-	2	-	-	1
Incorrect statements	4	19	11	7	11
Empty	88	26	43	5	38
Total	100	100	100	100	100

In Table 7, it shows that 38% of the pre-service teachers left unanswered the question on the division of a mixed number by another. Eighty-eight percent of first year students, 43% of third, 26% of second and 5% of fourth year students left this question unanswered. Moreover, 17% of the respondents posed simple exercises, while 11% posed problems requiring the application of proportions. Thirty-eight percent of fourth, 17% of second, 11% of third, and 2% of first year students posed problems as simple exercises. Proportion-based problems were used mostly by second year (19%) students, while first year (%2) students used them the least. Ten percent of participants had confusion with problems based on multiplication: 23% of fourth, 7% of third, and 4% of first and second year students had such problems. Furthermore, 9% posed problems in the *Measurement* category, while 11% posed problems with incorrect statements. Nineteen percent of second year students used incorrect statements, followed by 11% of third, 7% of fourth, and 4% of first year students. The only pre-service teacher to pose a problem using a diagram was a second year student (see Figure 1). This student used figures to denote the fractions, and asked for the performance of the operation to be completed.



(Perform the operations expressed by writing the dotted parts of the shapes.)

Figure 1. Problem posed with the help of a diagram

While the first, second and third year students in the study had very high rates of unanswered for this question, the fourth year students exhibited very low rates; instead they mostly employed simple exercises. However, with that said, the fourth year students exhibited confusion with multiplication, while the other classes did not do so to such a great extent.

Discussion, Conclusion, and Recommendations

This study has aimed to analyze the problems posed for multiplication and division operations with fractions by pre-service elementary mathematics teachers who were at different years of training, and to shed light on how their skills and understanding in these areas change. In this context, the pre-service teachers were asked to pose problems that reflected certain multiplication and division operations. The problems posed by the pre-service teachers were analyzed by examining how these problems were reflected in multiplication and division operations with fractions. The study results indicate that, regardless of the year of enrollment, the pre-service teachers in the program were more successful in posing problems related to multiplication than in posing problems related to division. This finding is consistent with Isik's (2011) study on fourth year pre-service elementary mathematics teachers. At the same time, Julie (2017) noted that the mathematics ability of teachers to calculate addition, subtraction, multiplication, and division of fractions was good. But the mathematics ability of teachers that really needs to be improved, specially multiplication and division of two fractions (Julie, 2017). The rates at which questions

were left empty, or the participants failed to pose problems, were much higher for the division related questions compared to those involving multiplication. It is likely that the pre-service teachers had more difficulty in posing problems with division compared to those with multiplications and therefore left a higher number of division questions unanswered, a result perhaps attributable to their inability to associate the division operation with their daily lives. There are certain studies that have confirmed this, reporting that pre-service teachers struggled to associate division problems with real-life cases (Ball, 1990; Toluk-Ucar, 2009). Another possible reason behind the difficulties pre-service teachers had with division problems could be related to their inexperience with problem posing in general. For example, Xie & Masingila (2017) noted in their study that some of the difficulties were likely due to the lack of experience prospective primary teachers had in problem posing, while others were due to their conceptual or operational misunderstanding of fractions. At the same time division was the most difficult operation for posing story problems (Xie & Masingila, 2017). It was found in the present study that the first year students left the most questions empty that dealt with division operations, whereas the fourth year students often had confusions with multiplications, or posed problems in the form of simple exercises. The cause behind the finding showing the first year students to be the ones who most frequently left questions unanswered could be connected to their lack of problem-posing experience in secondary education and the lack of having a conceptual understanding of fractions. Moreover, in only a very few instances did the pre-service teachers pose problems using diagrams.

It was observed that the pre-service teachers were generally successful in applying meaning to operations and numbers involved in the problems posed for the multiplication of fractions. For the first operation which involved multiplication of an integer with a proper fraction ($\frac{3}{7}x8$), most of the problems posed fell under the category of *Repeated additions*. The frequent use of repeated additions can be explained by the fact that it is shorthand for multiplication. Furthermore, in the problem posing process for multiplication of an integer with a proper fraction, the pre-service teachers focused solely on the operation, disregarding its result, and thus failed to convey a relationship between the parts and the whole through the problems. Isik (2011) notes similar findings. Fischbein, Deri, Nello & Marino (1985) noted that the repeated addition method is the primitive intuitive model of multiplication. In the case of the operation involving the multiplication of two proper fractions ($\frac{3}{4}x\frac{5}{7}$), the pre-service teachers usually posed problems asking to "calculate $\frac{5}{7}$ of $\frac{3}{4}$ of a whole" without specifying the whole. In the problem posing process of the multiplication of proper fractions, it was determined that pre-service teachers were unable to transfer the expression to the problem expression, which requires the application of the second fraction to the new fraction generated after applying the first whole fraction or the first fraction of the original fraction. Another consequence of the multiplication in simple fractions is that the pre-service teachers should pose problems that aim to find the number of parts that occur in the resultant product, ignoring the fact that the sum of the product is smaller than the multiplicities. The last multiplication operation offered in the survey involved the multiplication of a mixed number with a fraction equal to a quarter ($1\frac{1}{3}x\frac{1}{4}$). In response to this question, the pre-service teachers participating in the study most frequently posed problems that involved the "calculation of $\frac{1}{4}$/one fourth of $1\frac{1}{3}$" . The pre-service teachers were found to have difficulties with the multiplication of a mixed number with a quarter, as this was the question that the highest number of respondents chose to leave unanswered. This was probably caused by their failure to come up with a meaningful representation of mixed numbers in daily life (Isik, 2011). On the basis of these results, it can be said that, in general, pre-service teachers tend to ignore the conceptual meaning and quantity specifying feature (Armstrong, & Bezuk, 1995; Mack, 1998; Pagni, 1998).

In the case of multiplication operations, simple exercises were the most frequent choice of fourth year students, while the first year students were found to have left the most answers to the questions empty. Nearly all of the fourth year students answered each of the questions on the problem posing test. The experience with fractions that the pre-service teachers had gained throughout the course of the program at the faculty of education may have had something to do with the ability of the fourth year students to pose a problem for all the operations. On the other hand, lacking such experience, the first year students struggled with the concept of fractions. As was the case in Isik's (2011) study, pre-service teachers did, from time to time, pose problems where the result of the multiplication with proper fractions was larger than the multipliers. Yet, in the case of multiplication of two proper fractions, the result must be smaller than individual multipliers. The errors the pre-service teachers made may be related to a subconscious mindset about multiplication, where "the result is larger than the multipliers", as is the case in the multiplication with natural numbers. This tendency is probably reinforced by weaknesses in the conceptual understanding of fractions.

Pre-service teachers often had difficulties in attaching meaning to the operations and numbers, as evidenced by the problems they posed for division with fractions. In the case of the operation involving the division of an integer with a proper fraction ($5 \div \frac{5}{8}$), the pre-service teachers usually failed to pose a problem and left the question unanswered. The second largest category under which the problems posed by the pre-service teachers fell was *Measurement* –a different definition of division with fractions. The second operation in this context was the division of a proper fraction by another ($\frac{11}{12} \div \frac{1}{4}$). This question was mostly unanswered by the pre-service teachers. The measurement function was a difficult one to formulate in a problem involving the division of two simple fractions, as shown by the focus on equal sharing in the problems posed as simple exercises. However, for fractions, the meaning of measurement for division is more appropriate than the meaning of equal sharing (Ball, 1990; Ma, 1999). Different studies concur that the model most widely used by students and teachers with the division operation was that of equal sharing (Ball, 1990; Tirosh & Graeber, 1991). The most number of questions left answered was recorded in the last question, which involved the division of a mixed number by another mixed number ($3\frac{4}{5} \div 1\frac{1}{10}$). In this

operation, the pre-service teachers were observed to first convert the mixed number to a compound fraction, and then attach the meaning of a whole to a compound fraction, which ended up actually representing something larger than the whole. Furthermore, the respondents were observed to pose problems based on proportion, or to confuse division with multiplication, and it was also determined that pre-service teachers had difficulties in terms of fraction units. These results of the study are in parallel with the results of the study by Isik (2011).

Division with fractions has reference to two distinct meanings: measurement, and allocation of equal shares. However, the pre-service teachers who took part in the study had a preference for allocation of equal shares, rather than the measurement function. Furthermore, the pre-service teachers posed problems requiring division by the number in the denominator of the divisor fraction. Such findings of the study offer parallels with the conclusions drawn from other studies in the literature (Isik, 2011; Ma, 1999). The most striking difficulty experienced with division involved cases where a mixed number was to be divided by another. Some of the pre-service teachers tried to pose a problem where the mixed number was converted to a compound fraction. However, by doing so, they tried to impose the meaning of a unit onto the fraction, as Isik (2011) noted the same in his study. Teachers and pre-service teachers can achieve relative freedom from the textbooks in the problem solving processes in their own classes, only if they gain the ability to pose problems. If they have difficulties in problem posing, they would be limited to using the examples provided in the text books. The ability to pose problems for fractions, on the other hand, requires a conceptual learning of fractions. It is evident that, regardless of the year in the program, pre-service teachers have difficulties with divisions and multiplications with fractions, a shortcoming that could be related to the limited usage of fractions in daily life. Most of the problems observed were reflections of conceptual issues. This raises some questions about the effectiveness of teacher education programs in developing pre-service teachers' mathematical competence, as some studies have shown that the mathematics courses taken by pre-service teachers in their education program do not add too much to their mathematics knowledge they bring from secondary schooling (Toluk-Ucar, 2009).

Failure to remedy the conceptual weaknesses observed in this study might have significant repercussions on the education teachers will provide to their students. The results of this study also point to the necessity of eliminating the conceptual deficiencies of multiplication and division in fractions demonstrated by pre-service teachers. If they are not eliminated, the teachers will have difficulties in classroom teaching activities and in evaluating the problems posed by their students. Accordingly, it is important to identify the difficulties pre-service teachers have with these mathematical conceptions. In this context, education faculties have to take necessary measures to avoid these difficulties. The results of this study suggest that mathematics methods courses can be used to revise pre-service teachers' mathematical knowledge and at the same time different instructional strategies, such as problem posing, can help us to revise pre-service teachers' knowledge and beliefs about mathematics into these courses. Pre-service teachers need to consider teachers' suggestions on desirable attributes and practices that can help them achieve positive teaching experiences (Hudson, 2013). For this in these courses teachers and pre-service teachers can study together. Future studies should aim to determine the specific measures that can be taken for these difficulties. As a further recommendation, these future studies should involve interviews with pre-service teachers, and investigate, in depth, the issues they face in formulating problems, and the causes which lead to more significant difficulties with divisions with fractions, compared to those faced in the context of multiplication.

Acknowledgements

I would like to thank the pre-service teachers who participated in this study.

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