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Extremal Properties of a Certain Subclass of Spiral-Like Functions

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Abstract— In this article, we investigate the fundamental extremal and topological properties of a certain subclass of spiral-like functions. We establish that this subclass forms a compact subset within the locally convex linear topological space of analytic functions in the unit disk. Moreover, we derive membership relations and integral representation formulas for functions belonging in this subfamily. Furthermore, the findings extend and generalize previously known results, providing deeper insights into the structural properties of these functions.

Keywords — *Univalent functions, spiral-like functions, integral representation formulas*

Mathematics Subject Classification (2020) 30C45, 30C55

1. Introduction

Let $\mathcal{D}$ be the unit disc $\{z \in \mathbb{C} : |z| < 1\}$, where $\mathbb{C}$ denotes the set of all complex numbers, and $\mathcal{A}$ be the class of functions analytic in $\mathcal{D}$, satisfying the conditions $f(0) = 0$ and $f'(0) = 1$. Then, each $f \in \mathcal{A}$ has the Taylor expansion

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

$f \in \mathcal{A}$ is also univalent, we say that f belongs to class S.

The class P consists of all analytic functions p having a positive real part in $\mathcal{D}$ with $p(0) = 1$. The class P is also called the Carathedory Class. For an analytic function, we call it ϑ-spiral-like-function provided that its range is θ-spiral-like. Besides, $\theta-$spirals are the curves with constant radial angle θ with increasing modulus. The family of θ-spiral-like functions is stand by $\mathcal{S}_\theta$. Špaček [1] has defined the spiral-like functions. Analytically, $f \in \mathcal{A}$ belongs to the family $\mathcal{S}_\theta$ if and only if

$$\operatorname{Re}\left(e^{i\theta}\frac{zf'(z)}{f(z)}\right) > 0, \quad |\theta| < \frac{\pi}{2} \tag{1.2}$$

For $0 \leq \alpha < 1$, $S^*(\alpha)$ denotes the family of starlike functions of order α if and only if $\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \alpha$, for all $z \in \mathcal{D}$. Note that $S^*(0) = S^*$ is the class of starlike functions. Libera [2] has extended the classes of starlike functions of order α, $S^*(\alpha)$, and $\mathcal{S}_\theta$ by introducing a more general definition and thus broadened their scope and applicability in geometric function theory. In this context, the author has proposed the class of the normalized analytic functions in $\mathcal{A}$ satisfying the condition.

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$$\operatorname{Re}\left(e^{i\theta}\frac{zf'(z)}{f(z)}\right) > \alpha\cos\theta$$

for all $z \in \mathcal{D}$. Here, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\alpha \in [0, 1)$. This class is denoted by $\mathcal{S}_\theta(\alpha)$. Thus, the equalities $S_0(\alpha) = S^*(\alpha)$ and $\mathcal{S}_\theta(0) = \mathcal{S}_\theta$ are valid.

Recently, it has been an interesting problem for many authors to study spiral-like functions. Other interesting results on functions in certain subclasses of spiral-like functions can be investigated from [3–17]. In this study, we propose the generalized subclass of spiral-like functions denoted by $\mathcal{S}_\theta(\alpha, \lambda)$, the subclass of $\mathcal{A}$ consisting of functions of the form in (1.1) and satisfying the condition

$$\operatorname{Re}\left(e^{i\theta}\frac{zf'(z)}{(1-\lambda)f(z) + \lambda zf'(z)}\right) > \alpha\cos\theta \tag{1.3}$$

where $z \in \mathcal{D}$, $0 \le \lambda < 1$, $0 \le \alpha < 1$, and $|\theta| < \frac{\pi}{2}$. Note that

i. If $\lambda = 0$, then $\mathcal{S}_\theta(\alpha, 0) = \mathcal{S}_\theta(\alpha)$ studied by Libera [2]

ii. If $\lambda = 0$ and $\alpha = 0$, then we obtain the class of spiral-like functions defined by Specek [1]

iii. If $\lambda = 0$ and $\theta = 0$, then we obtain the class of starlike functions of order α

The rest of the paper is organized as follows: In Section 2, we present some basic definitions and properties to be needed in the next sections. In Section 3, we explore the extremal properties of $\mathcal{S}_\theta(\alpha, \lambda)$. In Section 4, we establish necessary and sufficient conditions and derive an integral representation formula for functions belonging to this subclass. Finally, we discuss the need for further research.

2. Preliminaries

Let $\mathcal{A}$ be the set of analytic functions of the form in (1.1). Then, $\mathcal{A}$ is locally convex linear topological space with the uniform convergence on a compact subset of $\mathcal{D}$. Let $\mathcal{F}$ be any subset of $\mathcal{A}$. Then, the intersection of all convex sets containing $\mathcal{F}$ is said to be the closed convex hull of $\mathcal{F}$ and denoted by $\overline{\text{co}}\mathcal{F}$. It can be observed that $\overline{\text{co}}\mathcal{F}$ consists of all elements of the form $\sum\limits_{k=1}^{m} l_k f_k$ where $f_k \in \mathcal{F}$, $l_k \ge 0$, and $\sum\limits_{k=1}^{m} l_k = 1$. Let the set $\mathcal{F}$ be a compact subset of $\mathcal{A}$. A function f is called an extreme point of $\mathcal{F}$ if $f \in \mathcal{F}$ and

$$f = lf_1 + (1-l)f_2 \text{ implies } f = f_1 = f_2$$

where $0 < l < 1$ and f_1, $f_2 \in \mathcal{F}$. It is convenient to use the $\mathcal{EF}$ notation for the set of extreme points of $\mathcal{F}$. Sharp upper bounds for non-linear functionals can be obtained using the set of extreme points. This approach is based on Theorem 2.1, called the Krein-Milman theorem.

Theorem 2.1. [18] A compact convex subset of a Hausdorff locally convex topological space equals the closed convex hull of its extreme points.

Moreover, the Montel's theorem to be used in this study is also as follows:

Theorem 2.2. [19] If $\mathcal{F}$ is a family of analytic functions defined on an open set $\Omega \in \mathbf{C}$, uniformly bounded on every compact subset of Ω, then $\mathcal{F}$ is normal.

If $f \in \mathcal{S}_\theta$, then, from (1.2),

$$\operatorname{Re}\left\{e^{i\theta}\frac{zf'(z)}{f(z)}\right\} > 0, \quad |\theta| < \frac{\pi}{2}$$

Consider the function

$$f(z) = \frac{z}{(1-z)^{2\tau}}, \ \tau = \cos\theta e^{-i\theta}, \ \ |\theta| < \frac{\pi}{2}$$
$$= \frac{z^{1-\tau}}{4^{\tau}} \ \left[\left(\frac{1+z}{1-z}\right)^2 - 1\right]$$

This function maps $\mathcal{D}$ onto the complement of an arc of $\theta-$spiral. By suitable choice of arguments, the condition in (1.2) is equivalent to

$$0 < \arg\left\{ ie^{i\theta}\left[\frac{re^{i\theta} f'\left(re^{i\theta}\right)}{f\left(re^{i\theta}\right)}\right]\right\} < \pi$$

That is, the image of the circle $|z| = r$, $0 < r < 1$, under f is the curve C_r given by $w = f\left(re^{i\theta}\right)$ and its radial angle is $\arg\left\{i\frac{zf'(z)}{f(z)}\right\}$. Following representation formula has been obtained by Libera [2] for functions belonging to the class $\mathcal{S}_\theta$.

Lemma 2.3. [2] $f \in \mathcal{S}_\theta(\alpha)$ if and only if there exists a probability measure μ on $\partial\mathcal{D}$ such that

$$f(z) = z\exp\left(\int\limits_{|x|=1} -2\tau\log(1-xz)\,d\mu(x)\right), \quad \tau = (1-\alpha)\cos\theta e^{-i\theta} \tag{2.1}$$

The relation between the set of probability measures on $\partial\mathcal{D}$ and $\mathcal{S}_\theta(\alpha)$ given by (2.1) is one-to-one.

Lemma 2.4. [20] Let $\mathcal{P}$ the class of functions $p(z)$ with positive real part and $p(0) = 1$ in $\mathcal{D}$. Then, $p \in \mathcal{P}$ if and only if there exists a probability measure μ on $\partial\mathcal{D}$ such that

$$p(z) = \int\limits_{|x|=1} \frac{1+xz}{1-xz} d\mu(x), \quad |z| < 1 \tag{2.2}$$

Lemma 2.5. [21] Let Λ denotes the set of all probability measures on $\partial\mathcal{D}$. Then, the set of extreme points Λ consists of the points masses.

Lemma 2.6. [21] $p \in \mathcal{P}$ if and only if there exists a sequence of functions $\{p_n\}$ such that

$$p(z) = \sum_{k=1}^{m} t_k \frac{1+x_k z}{1-x_k z} \tag{2.3}$$

where $|x_k| = 1$, $t_k \geq 0$, $\sum\limits_{k=1}^{m} t_k = 1$, and $p_n \to p$ uniformly on compact subset of $\mathcal{D}$.

3. Extremal Structure of the Class $\mathcal{S}_\theta(\alpha, \lambda)$

This section explores the extremal properties of a certain subclass of spiral-like functions.

Theorem 3.1. The class of spiral-like functions $\mathcal{S}_\theta(\alpha, \lambda)$ is a compact subset of locally convex topological space $\mathcal{A}$.

Proof. Since $\mathcal{S}_\theta(\alpha, \lambda)$ is a compact subset S of $\mathcal{A}$, it is enough to show that $\mathcal{S}_\theta(\alpha, \lambda)$ is closed. Suppose $f_n \in \mathcal{S}_\theta(\alpha, \lambda)$ and that $f_n \to f$ uniformly on compact subset of $\mathcal{D}$. Then, from (1.3),

$$\mathrm{Re}\left\{\frac{e^{i\theta}\sec\theta\left(\frac{zf'(z)}{(1-\lambda)f(z)+\lambda zf(z)}\right) - i\tan\theta - \alpha}{1-\alpha}\right\} = \mathrm{Re}\{g(z)\} \geq 0 \tag{3.1}$$

It is only enough to show that the strict inequality holds in (3.1), for all $z \in \mathcal{D}$. If $g(z)$ is not constant, Then, the open mapping theorem can be observed. If it is constant, then since $f \in S$, it should be $f(z) = z$ and from the condition $|\theta| < \frac{\pi}{2}$,

$$\mathrm{Re}\left\{\frac{e^{i\theta}\sec\theta\left(\frac{zf'(z)}{(1-\lambda)f(z)+\lambda zf(z)}\right) - i\tan\theta - \alpha}{1-\alpha}\right\} = \cos\theta > 0$$

□

Then, we provide the following integral representation of functions in the class $\mathcal{S}_\theta(\alpha,\lambda)$.

Theorem 3.2. $f \in S_\theta(\alpha,\lambda)$ if and only if there is a probability measure on $\partial\mathcal{D}$ such that

$$f(z) = z\exp\left[\int\limits_{|x|=1} -\frac{2\tau}{1+2\lambda}\log\left(1-(1+2\lambda)xz\right)d\mu(x)\right] \tag{3.2}$$

where $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$. The correspondence from the set of probability measures on $\partial\mathcal{D}$ to $S_\theta(\alpha,\lambda)$ given by (3.2) is one to one.

Proof. Let $f \in S_\theta(\alpha,\lambda)$. Then, the mapping from $\mathcal{S}_\theta(\alpha,\lambda)$ to $\mathcal{P}$ defined by

$$p(z) = \frac{e^{i\theta}\sec\theta\left(\frac{zf'(z)}{(1-\lambda)f(z)+\lambda zf(z)}\right) - i\tan\theta - \alpha}{1-\alpha} \tag{3.3}$$

is one to one correspondence from $\mathcal{S}_\theta(\alpha,\lambda)$ onto $\mathcal{P}$. Since f is analytic in $\mathcal{D}$, $f(0) = 0$, $f'(0) = 1$, and $p \in \mathcal{P}$, where p is given by (3.3), it follows that $f(z) \neq 0$ when $z \neq 0$. Thus,

$$g(z) = \log\left(\frac{f(z)}{z}\right)$$

is well defined and analytic in $\mathcal{D}$ with the choice of the branch so that $g(0) = 0$. Since

$$\frac{d}{dz}\{g(z)\} = \frac{\tau h(z)}{(1+\lambda)-\lambda p(z)}$$

where $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$ and $h(z) = \frac{p(z)-1}{z}$, then

$$g(z) = \tau\int\limits_0^z \frac{p(w)-1}{w}\frac{dw}{(1+\lambda)-\lambda p(w)}$$

and

$$f(z) = z\exp\left\{\tau\int\limits_0^z \frac{p(w)-1}{w}\frac{dw}{(1+\lambda)-\lambda p(w)}\right\}$$

By using representation given by (2.2),

$$\begin{aligned} f(z) &= z\exp\left\{\tau\int\limits_{|x|=1}\left\{\int\limits_0^z \frac{2x}{1-xw}\frac{(1-xw)\,dw}{1-(1+2\lambda)p(w)}\right\}d\mu(x)\right\} \\ &= z\exp\left\{\int\limits_{|x|=1} -\frac{2\tau}{1+2\lambda}\log\left(1-(1+2\lambda)xw\right)d\mu(x)\right\} \end{aligned}$$

where $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$. The one to one correspondence between $\mathcal{S}_\theta(\alpha,\lambda)$ and $\{\mu\}$ on $\partial\mathcal{D}$ given by (3.2) may be viewed as equivalent to a uniqueness statement about moments. Suppose p and q belong to $\mathcal{P}$ defined by (3.3) and correspond to the measure μ and ν given by (3.2), respectively. If

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n \text{ and } q(z) = 1 + \sum_{n=1}^{\infty} q_n z^n$$

then it can be observed that $p \in \mathcal{P}$ if and only if $p(z) = \int\limits_{|x|=1} \frac{1+xz}{1-xz} d\mu(x)$ implies that

$$p_n = 2 \int\limits_{|x|=1} x^n d\mu(x) \text{ and } q_n = 2 \int\limits_{|x|=1} x^n d\nu(x), \quad \text{for } n \in \{1, 2, \ldots\}$$

Hence, $p = q$ implies that if μ and ν are two probability measures on $\partial\mathcal{D}$ such that

$$\int\limits_{|x|=1} x^n d\mu(x) = \int\limits_{|x|=1} x^n d\nu(x), \quad \text{for } n \in \{1, 2, \ldots\}$$

then $\mu = \upsilon$ and conversely. Thus, the correspondence from the set of probability measures on $\partial\mathcal{D}$ to $\mathcal{S}_\theta(\alpha, \lambda)$ given by (3.2) is one to one. □

In the case of $\lambda = 0$, it is possible to obtain an integral formula given in (2.1), derived by Libera [2] for functions in the class $\mathcal{S}_\theta(\alpha)$. Moreover, it is possible to uniformly approximate for a function $f \in \mathcal{S}_\theta(\alpha, \lambda)$ on $|z| \le r$ by functions given in the following theorem.

Theorem 3.3. $f \in \mathcal{S}_\theta(\alpha, \lambda)$ if and only if there is a squence of functions $\{f_n\}$ having form

$$f_n(z) = z \exp \left\{ 2\tau \int_0^z \sum_{k=1}^{n} \frac{t_k x_k}{1 - x_k w} \frac{dw}{\sum\limits_{k=1}^{n} t_k \left(\frac{1-(1+2\lambda)x_k w}{1-x_k w} \right)} \right\} \tag{3.4}$$

where $|x_k| = 1$, $t_k \ge 0$, $\sum\limits_{k=1}^{m} \tau t_k = \tau$, $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$, $|\theta| < \frac{\pi}{2}$, and $f_n \to f$ uniformly on compact subset of $\mathcal{D}$.

Proof. According to Theorem 3.2, $f \in \mathcal{S}_\theta(\alpha, \lambda)$ if and only if

$$f(z) = z \exp \left[\int\limits_{|x|=1} -\frac{2\tau}{1+2\lambda} \log\left(1 - (1+2\lambda)xz\right) d\mu(x) \right]$$

such that $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$. Since μ is a probability measure υ_n, convex combinations of point masses so that

$$\int\limits_{|x|=1} g(x) d\upsilon_n(x) \to \int\limits_{|x|=1} g(x) d\mu(x)$$

for every continuous function g on $\partial\mathcal{D}$. The proof of this expression depends on Theorem 2.1, (2.3) in Lemma 2.6, and the fact that the set of probability measures on $\partial\mathbb{D}$ is compact in weak star topology. Thus,

$$\upsilon_n = \sum_{k=1}^{m} t_k \delta_{xk} \quad \text{where} \quad t_k \ge 0 \quad \text{and} \quad \sum_{k=1}^{m} t_k = 1$$

and δ_{xk} denotes the point mass at x_k. If f_n denotes the functions in $\mathcal{S}_\theta(\alpha, \lambda)$ corresponding to measures υ_n, then it follows that $f_n \to f$ uniformly on compact subset of $\mathcal{D}$. The uniform convergence follows from Montels's theorem, i.e., Theorem 2.2, and the facts that

$$|f_n(z)| \le \max_{|x|=1} \frac{|z|}{|1 - (1+2\lambda)\, xz|^{\frac{2\tau}{1+2\lambda}}} \quad \text{and} \quad |f_n(z)| \le \max \frac{|z|}{|1 - (1+2\lambda)\, xz|^{\frac{2\tau}{1+2\lambda}}}$$

such that $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$. Besides, since $f_n \in S_\theta(\alpha, \lambda)$,

$$f_n(z) = z \exp\left\{\tau \int_0^z \frac{p_n(w)-1}{w} \frac{dw}{(1+\lambda) - \lambda p_n(w)}\right\}$$

$p_n(w) \in \mathcal{P}$ and $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$. Applying the relation in (2.3) in Lemma 2.6, for $p_n \in \mathcal{P}$,

$$f_n(z) = z \exp\left\{2\tau \int_0^z \sum_{k=1}^{n} \frac{t_k x_k}{1 - x_k w} \frac{dw}{\sum\limits_{k=1}^{n} t_k \left(\frac{1-(1+2\lambda)x_k w}{1-x_k w}\right)}\right\}$$

where $|x_k| = 1$, $t_k \geq 0$, $\sum\limits_{k=1}^{m} 2\tau t_k = 2\tau$, and $\tau = \frac{(1-\alpha)\cos\theta e^{-i\theta}}{1-\lambda}$. The converse follows from Theorem 3.3 and the fact that $f_n(z)$ given by (3.4) is in $\mathcal{S}_\theta(\alpha, \lambda)$. □

For $\lambda = 0$, the following result can be observed:

Corollary 3.4. $f \in \mathcal{S}_\theta(\alpha)$ if and only if there is a sequence of functions $\{f_n\}$ having form

$$f_n(z) = \frac{z}{\prod\limits_{k=1}^{n} (1 - x_k z)^{2\tau t_k}}$$

where $|x_k| = 1$, $t_k \geq 0$, $\sum\limits_{k=1}^{n} 2\tau t_k = 2\tau$, $\tau = e^{-i\theta}\cos\theta$, $|\theta| \leq \frac{\pi}{2}$, and $f_n \to f$ uniformly on compact subset of $\mathcal{D}$.

4. Necessary and Sufficient Condition for the Class $\mathcal{S}_\theta(\alpha, \lambda)$

The integral representation formula in (3.2) of Theorem 3.2 can be considered a necessary and sufficient condition. This section proposes an alternative case distinct from (3.2).

Theorem 4.1. Let $0 \leq \lambda < 1$, $\alpha \in [0, 1)$, $|\theta| < \frac{\pi}{2}$, and $f(z)$ be of the form in (1.1). Then, $f \in \mathcal{S}_\theta(\alpha, \lambda)$ if and only if the following condition holds:

$$\sum_{k=1}^{\infty} \left\{(k-1)(1 - \lambda(\alpha + i\tan\theta)) + 2e^{2i\theta} - \lambda(1-\alpha)\left(1 - e^{2i\theta}\right)(k-1)\right\}(1 + (k-1)\lambda) a_k z^k \neq 0$$

$z \in \mathcal{D} \backslash \{0\}$ and $k \in \mathbb{N} \backslash \{1\}$, where the notation $\mathbb{N}$ represents the set of all positive integers.

Proof. Consider the function

$$p(z) = \frac{e^{i\theta}\sec\theta\left(\frac{f(z)}{(1-\lambda)f(z)+\lambda z f'(z)}\right) - i\tan\theta - \alpha}{1-\alpha}$$

Thus, p is an analytic function, satisfying $p(0) = 1$ and $\operatorname{Re}(p(z)) > 0$. Then, $f \in \mathcal{S}_\theta(\alpha, \lambda)$ if and only if

$$p(z) \neq \frac{1 - e^{2i\theta}}{1 + e^{2i\theta}}$$

Hence,

$$\frac{e^{i\theta}\sec\theta z f'(z) - (\alpha + i\tan\theta)((1-\lambda) f(z) + \lambda z f'(z))}{(1-\alpha)((1-\lambda) f(z) + \lambda z f'(z))} \neq \frac{1 - e^{2i\theta}}{1 + e^{2i\theta}}$$

By using the series expansion of $f(z)$, given by (1.1),

$$(1 + e^{2i\theta}) \sum_{k=1}^{\infty} [(k-1)(1 - \alpha\lambda - i\lambda\tan\theta) + (1-\alpha)](1 + (k-1)\lambda) a_k z^k \neq (1-\alpha)\left(1 - e^{2i\theta}\right) \sum_{k=1}^{\infty} (1 + (k-1)\lambda)^2 a_k z^k$$

for $z \neq 0$. It is equivalent to

$$\sum_{k=1}^{\infty} \left\{ (k-1)(1-\lambda(\alpha+i\tan\theta)) + 2e^{2i\theta} - (1-\alpha)\left(1-e^{2i\theta}\right)(k-1)\lambda \right\} (1+(k-1)\lambda) a_k z^k \neq 0$$

□

5. Conclusion

This article explores the extremal and topological properties of a specific subclass of spiral-like functions. Our results demonstrate that this subclass forms a compact subset within the locally convex linear topological space of analytic functions in the unit disk. In addition, we derived membership relations and an integral representation formula for functions within this subfamily. These findings extend existing results and offer new insights into the structural behavior of spiral-like functions. Using the results obtained in this paper, sharp upper bounds can be determined for the Fekete-Szegö functional and the Hankel determinant for functions in the class $S_\theta(\alpha, \lambda)$. The technique used in this paper can also be applied to the class of Bazilevič functions, a broader class than spiral-like functions.

Author Contributions

The author read and approved the final version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

Ethical Review and Approval

No approval from the Board of Ethics is required.

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