MATHEMATICAL SCIENCES AND APPLICATIONS E-NOTES
VOL:4 (1) 40-44 (2016) ©MSAEN

On Some Special Codes Over $\mathbb{F}_3 + v\mathbb{F}_3 + u\mathbb{F}_3 + u^2\mathbb{F}_3$

Mustafa ÖZKAN, Figen ÖKE*

(Communicated by Murat TOSUN)

Abstract
In this paper the structure of the ring $\mathbb{F}_3[u,v]/_{< u^3,\ v^2, uv >}$ where $u^3 = 0$,$v^2 = 0$ and $uv = vu = 0$ is described. The distance function on the ring $R = \mathbb{F}_3 + v\mathbb{F}_3 + u\mathbb{F}_3 + u^2\mathbb{F}_3$ which is isomorphic to the ring is defined. This means that linear codes over the ring R can be written. Then it's shown that the Gray images of cyclic codes over the ring R are quasi-cyclic codes of index 2 over the ring $\mathbb{F}_3 + v\mathbb{F}_3$. Then another Gray map from the ring $\mathbb{F}_3 + v\mathbb{F}_3$ to $\mathbb{F}_3$ is described. Thus the relation between the cyclic codes over the ring $R = \mathbb{F}_3 + v\mathbb{F}_3 + u\mathbb{F}_3 + u^2\mathbb{F}_3$ and quasi-cyclic codes over field $\mathbb{F}_3$ is established.

Keywords: *Linear Codes; Quasi-Cylic Codes; Gray Map; Codes Over Rings.*

AMS Subject Classification (2010): *Primary: 94B05 ; Secondary: 94B15; 94B60.*

****Corresponding author***

1. Introduction

Cyclic codes, constacyclic codes over different rings (Finite chain rings, Frobenious rings, i.e.) were studied before. Also the Gray images of these codes and structure of codes over fields are discussed before. Previous years the relations between codes over the rings in one variable and the codes over fields were studied. In recent years the relations between the codes over the rings in more variables and the codes over fields have been studied. Especially in [9]; $(1 + v)$ -constacyclic codes over $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2 + uv\mathbb{F}_2$ where $u^2 = v^2 = 0$, $u.v - v.u = 0$ were studied by S. Karadeniz and B. yıldız. In [10]; X. Xiaofang studied $(1 + v)$ -constacyclic codes over the ring $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2$ where $u^2 = v^2 = 0$, $u.v = v.u = 0$. In this study a special ring in two variables is defined under certain conditions. The structure of the cyclic codes over this ring is investigated. Relations between the codes over this ring and the codes over a finite chain ring in one variable are obtained via a Gray map. Another Gray map from the finite chain ring to a finite field is defined and then the images of quasi-cyclic codes are obtained. Consequently; ıt isshown that the cyclic codes over the ring R are the quasi-cyclic codes over field $\mathbb{F}_3$ via the composite of two new Gray maps.

2. Preliminaries

It is known that $R_1 = \mathbb{F}_3 + u\mathbb{F}_3 + u^2\mathbb{F}_3$ where $u^3 = 0$ and $R_2 = \mathbb{F}_3 + v\mathbb{F}_3$ where $v^2 = 0$ are the commutative rings. In the ring R_1; writing $R_2 = \mathbb{F}_3 + v\mathbb{F}_3$ where $v^2 = 0$ instead of $\mathbb{F}_3$, the set $R_2 + uR_2 + u^2R_2$ where $u^3 = 0$ is obtained. Then we have $R_2 + uR_2 + u^2R_2 = (\mathbb{F}_3 + v\mathbb{F}_3) + u.(\mathbb{F}_3 + v\mathbb{F}_3) + u^2.(\mathbb{F}_3 + v\mathbb{F}_3) = \mathbb{F}_3 + v\mathbb{F}_3 + u\mathbb{F}_3 + uv\mathbb{F}_3 + u^2\mathbb{F}_3 + u^2v\mathbb{F}_3$

Received : *05–August–2015,* ***Accepted :*** *14–February–2016*

This article is the written version of author's plenary talk delivered on September 03- August 31, 2015 at 4th International Eurasian Conference on Mathematical Sciences and Applications (IECMSA-2015) in Athens, Greece.

the condition $uv = vu = 0$ to the conditions $u^3 = 0$ in R_1 and $v^2 = 0$ in R_2 . Then the set $R_2 + uR_2 + u^2R_2$ is equal to the set $R = \mathbb{F}_3 + v\mathbb{F}_3 + u\mathbb{F}_3 + u^2\mathbb{F}_3$. Thus is a ring with the usual addition and multiplication under the conditions $u^3 = 0$,$v^2 = 0$, $uv = vu = 0$. It is easily seen that R is isomorphic to the ring $\mathbb{F}_3[u,v]/_{< u^3 = 0,\ v^2 = 0, uv = vu = 0 >}$.

Let C be a linear $[n,k,d]\_$ code. It means that C has the length n , it's dimention is k and it's minimum distance is d . Let R be a ring. Each submodule C of R^n is called a linear code with lenght n over the ring R. A subspace C of ${\mathbb{F}_3}^n$ is called a linear code with lenght n over the field $\mathbb{F}_3$. Each codeword c in such a code C is a n-tuple of the form $c = (c_o, c_1, ..., c_{n-1}) \in R^n$ and can be represented by $c = (c_1, c_2, ..., c_n) \longleftrightarrow c(x) = \sum_{i=1}^{n} c_i.x^i \in R[x]$. This notation can be written for the elements of ${R_1}^n$, ${R_2}^n$ and $\mathbb{F}_3^n$ similarly.

The Gray map from the ring R to R_1^2 is defined as ;

$$\Phi_1 : R \longrightarrow R_1^2$$

$$a + bv + cu + du^2 \mapsto \Phi_1(a + bv + cu + du^2) = \Phi_1(r + qv) = (2q, q + 2r)$$

where $r = a + cu + du^2$ and $q = b + au + (a + c)u^2$. Then we have

$\Phi_1(a + bv + cu + du^2) = (2b + 2au + (2a + 2c)u^2, (b + 2a) + (a + 2c)u + (a + c + 2d)u^2)$. The map Φ_1 can be generalized to R^n as ;

$\Phi_1(t_1, t_2, ..., t_n) = (q_1, q_2, ..., q_n, q_1 + r_1, q_2 + r_2, ..., q_n + r_n)$ where $t_i = r_i + q_i v$ such that $r_i = a_i + c_i u + d_i u^2$, $q_i = b_i + a_i u + (a_i + c_i)u^2$ for all $i = 1, 2, ..., n$. Note that the Gray map from R_1 to $\mathbb{F}_3^9$ is defined as ;

$$\Phi_2 : R_1 \longrightarrow \mathbb{F}_3^9$$

$$x + yu + zu^2 \mapsto \Phi_2(x + yu + zu^2)$$

here $\Phi_2(x + yu + zu^2) = (z, y + z, 2y + z, x + z, x + y + z, x + 2y + z, 2x + z, 2x + y + z, 2x + 2y + z)$.

The map Φ_2 can be generalized to R_1^n as ;

$$\begin{aligned}\Phi_2(b_1, b_2, ..., b_n) = {} & (z_1, z_2, ..., z_n, y_1 + z_1, y_2 + z_2, ..., y_n + z_n, 2y_1 + z_1, 2y_2 + z_2, ..., 2y_n + z_n, \\ & x_1 + z_1, x_2 + z_2, ..., x_n + z_n, x_1 + y_1 + z_1, x_2 + y_2 + z_2, ..., x_n + y_n + z_n, \\ & x_1 + 2y_1 + z_1, x_2 + 2y_2 + z_2, ..., x_n + 2y_n + z_n, 2x_1 + z_1, 2x_2 + z_2, ..., 2x_n + z_n, \\ & 2x_1 + y_1 + z_1, 2x_2 + y_2 + z_2, ..., 2x_n + y_n + z_n, 2x_1 + 2y_1 + z_1, 2x_2 + 2y_2 + z_2, ..., \\ & 2x_n + 2y_n + z_n)\end{aligned}$$

For each $(b_1, b_2, ..., b_n) \in R_1^n$, where $b_i = x_i + y_i u + z_i u^2$,$x_i, y_i, z_i \in \mathbb{F}_3$, for $i = 1, 2, ..., n$. The weight function w_R for each element r of R =$\mathbb{F}_3 + v\mathbb{F}_3 + u\mathbb{F}_3 + u^2\mathbb{F}_3$ is defined as ;

$$w_R(r) = \begin{cases} 12 & ; \ r \in R - R_1.u \\ 15 & ; \ r \in R_1.u - R_1.u^2 \\ 9 & ; \ r \in R_1.u^2 \\ 0 & ; \ r = 0 \end{cases}$$

Then $w_R(r) = \sum_{i=1}^{n} w_R(r_i)$ is satisfied for each element $r = (r_1, r_2, ..., r_n) \in R^n$. It is known that the homogeneous weight of each $s \in R_1$ is defined as ;

$$w_{\text{hom}}(s) = \begin{cases} 6 & ; \ s \in R_1 - R_1.u^2 \\ 9 & ; \ s \in R_1.u^2 - \{0\} \\ 0 & ; \ s = 0 \end{cases}$$

Then $w_{\text{hom}}(s) = \sum_{i=1}^{n} w_{\text{hom}}(s_i)$ is satisfied for each element $s = (s_1, s_2, ..., s_n) \in R_1^n$. The Hamming weight on $\mathbb{F}_3$ is defined as $w_H(0) = 0$, $w_H(1) = 1$, $w_H(2) = 1$. Hence $w_H(c) = \sum_{i=1}^{n} w_H(c_i)$ is hold for each $c = (c_1, c_2, ..., c_n) \in \mathbb{F}_3^n$.

The minimum distance of a code C is defined as ;

$d_R(C) = \min\{d_R(x,y)\}$, here $x, y \in C$, $x \neq y$ if C is a code over R ,

$d_{\text{hom}}(C) = \min\{d_{\text{hom}}(x,y)\}$, here$x, y \in C$, $x \neq y$if C is a code over R_1 and

$d_H(C) = \min\{d_H(x,y)\}$, here $x, y \in C$, $x \neq y$ if C is a code over $\mathbb{F}_3$. Each element of R is written as $a + bv + cu + du^2 = r + qv$ where $r = a + cu + du^2 \in R_1$, $q = b + au + (a+c)u^2 \in R_1$. It is clearly seen that the equalities $w_R(r) = w_{\text{hom}}(\Phi_1(r)) = w_H(\Phi_2(\Phi_1(r)))$ for each $r \in R^n$ are satisfied. Therefore it means that Φ_1 is an isometry from (R^n, d_R) to $(R_1^{2n}, d_{\text{hom}})$ and Φ_2 is an isometry from $(R_1^{2n}, d_{\text{hom}})$ to $(\mathbb{F}_2^{8n}, d_H)$. Expressing elements of R as $a + bv + cu + du^2 = r + qv$ where $r = a + cu + du^2 \in R_1$ and $q = b + au + (a+c)u^2 \in R_1$ are both in R_1 ,

we see that

$w_R(a + bv + cu + du^2) = w_R(r + qv) = w_{\text{hom}}(2q, q + 2r) = w_{\text{hom}}(2b + 2au + (2a+2c)u^2, (b + 2a) + (a+2c)u + (a+c+2d)u^2) = w_H(2a+2c, a+c+2d, a+2c, 2a+2d, 2c, 2c+2d, 2a+2b+2c, b+c+2d, a+2b+2c, a+b+2d, 2b+2c, 2a+b+2c+2d, 2a+b+2c, 2a+2b+c+2d, a+b+2c, 2b+2d, b+2c, a+2b+2c+2d)$

A cyclic shift on R^n is a permutation σ such that

$$\sigma(c_o, c_1, ..., c_{n-1}) = (c_{n-1}, c_0, ..., c_{n-2}) .$$

A linear code C over R of lenght n is said to be cyclic code if it is satisfied the equality $\sigma(C) = C$. Let $a \in R_1^{2n}$ with $a = (a_0, a_1, ..., a_{n-1}) = (a^{(0)} \left| a^{(1)} \right.)$, $a^{(i)} \in R_1^n$, for all $i = 0, 1$. Let $\sigma^{\otimes 2}$ be the map from R_1^{2n} to R_1^{2n} given by

$$\sigma^{\otimes 2}(a) = (\sigma(a^{(0)}) \left| \sigma(a^{(1)}) \right.)$$

where σ is the usual cyclic shift. A code D of lenght $2n$ over R_1 is said to be quasicyclic code of index 2 of $\sigma^{\otimes 2}(D) = D$. A cyclic shift on R_1^{2n} is a permutation τ such that

$$\tau(d_o, d_1, ..., d_{2n-1}) = (d_{2n-1}, d_0, ..., d_{2n-2}) .$$

A linear code D' over R_1 of lenght $2n$ is said to be cyclic code if it is satisfied the equality $\tau(D') = D'$. Let $a \in \mathbb{F}_3^{18n}$ with $a = (a_0, a_1, ..., a_{n-1}) = (a^{(0)} \left| a^{(1)} \right| a^{(2)} \left| ... \right| a^{(8)} \ )$, $a^{(i)} \in \mathbb{F}_3^n$, for all $i = 0, 1, ..., 8$. Let $\sigma^{\otimes 9}$ be the map from $\mathbb{F}_3^{18n}$ to $\mathbb{F}_3^{18n}$ given by

$$\sigma^{\otimes 9}(a) = (\sigma(a^{(0)}) \left| \sigma(a^{(1)}) \right| \sigma(a^{(2)}) \left| ... \right| \sigma(a^{(8)}) \ )$$

where σ is the usual cyclic shift. A code C' of lenght $18n$ over $\mathbb{F}_3$ is said to be quasicyclic code of index 9 of $\sigma^{\otimes 9}(C') = C'$.

3. Gray images of codes over the ring R

In this section firstly it will be shown that the Φ_1 Gray image of a cyclic code over R is a quasi-cyclic code of index 2 with even length. Secondly it will be shown that the Φ_2 Gray image of a cyclic code over R_1 is a quasi-cyclic code of index 9 with even length.

Lemma 3.1. *$\Phi_1.\sigma = \sigma^{\otimes 2}.\Phi_1$ is satisfied.*

Proof. Let $c = (c_0, c_1, ..., c_{n-1}) \in R^n$ where $c_i = r_i + q_i v$ for $0 <= i <= n-1$.

If $\Phi_1(c) = \Phi_1(c_0, c_1, ..., c_{n-1}) = \Phi_1(r_0 + q_0 v, r_1 + q_1 v, ..., r_{n-1} + q_{n-1} v). = (2q_0, 2q_1, ..., 2q_{n-1}, q_0 + 2r_0, q_1 + 2r_1, ..., q_{n-1} + 2r_{n-1})$ then $\sigma^{\otimes 2}(\Phi_1(c)) = (2q_{n-1}, 2q_0, ..., 2q_{n-2}, q_{n-1} + 2r_{n-1}, q_0 + 2r_0, ..., q_{n-2} + 2r_{n-2})$

On the other hand, $\sigma(c) = \sigma(c_0, c_1, ..., c_{n-1}) = (c_{n-1}, c_0, ..., c_{n-2})$.

Then $\Phi_1(\sigma(c)) = \Phi_1(\sigma(c)) = \Phi_1 \sigma(c_0, c_1, ..., c_{n-1}) = \Phi_1(c_{n-1}, c_0, ..., c_{n-2}) = \Phi_1(r_{n-1} + vq_{n-1}, r_0 + vq_0, ..., r_{n-2} + vq_{n-2}) = (2q_{n-1}, 2q_0, ..., 2q_{n-2}, q_{n-1} + 2r_{n-1}, q_0 + 2r_0, ..., q_{n-2} + 2r_{n-2})$.

□

Theorem 3.1. *A code C with length n over R is a cyclic code if and only if $\Phi_1(C)$ is a quasi-cyclic code of index 2 with length $2n$ over R_1 .*

Proof. Suppose that C is a cyclic code. Then $\sigma(C) = C$. By applying Φ_1, we have $\Phi_1(\sigma(C)) = \Phi_1(C)$. By using the Lemma 3.1, we have $\sigma^{\otimes 2}(\Phi_1(C)) = \Phi_1(\sigma(C)) = \Phi_1(C)$. So $\Phi_1(C)$ is a quasi-cyclic code of index 2 . Conversely, if $\Phi_1(C)$ is a quasi-cyclic code of index 2, then $\sigma^{\otimes 2}(\Phi_1(C)) = \Phi_1(C)$. By using the Lemma 3.1, we have $\sigma^{\otimes 2}(\Phi_1(C)) = \Phi_1(\sigma(C)) = \Phi_1(C)$. Since Φ_1 is injective then $\sigma(C) = C$.

□

Lemma 3.2. *$\sigma^{\otimes 9}.\Phi_2 = \Phi_2.\tau$ is satisfied.*

Proof. Let $t = (t_0, t_1, ..., t_{2n-1}) \in R_1^{2n}$ where $t_i = x_i + y_i u + z_i u^2$ for $0 <= i <= 2n-1$.
If $\tau(t) = \nu(t_0, t_1, ..., t_{2n-1})$ =$(t_{2n-1}, t_0, ..., t_{2n-2})$, then $\Phi_2.\tau(t) = \Phi_2(t_{2n-1}, t_0, ..., t_{2n-2})$

$$\begin{aligned}
&= (z_{2n-1}, z_0, ..., z_{2n-2}, y_{2n-1} + z_{2n-1}, y_0 + z_0, ..., y_{2n-2} + z_{2n-2}, 2y_{2n-1} + z_{2n-1},\\
&2y_0 + z_0, ..., 2y_{2n-2} + z_{2n-2}, x_{2n-1} + z_{2n-1}, x_0 + z_0, ..., x_{2n-2} + z_{2n-2},\\
&x_{2n-1} + y_{2n-1} + z_{2n-1}, x_0 + y_0 + z_0, ..., x_{2n-2} + y_{2n-2} + z_{2n-2},\\
&x_{2n-1} + 2y_{2n-1} + z_{2n-1}, x_0 + 2y_0 + z_0, ..., x_{2n-2} + 2y_{2n-2} + z_{2n-2},\\
&2x_{2n-1} + z_{2n-1}, 2x_0 + z_0, ..., 2x_{2n-2} + z_{2n-2}, 2x_{2n-1} + y_{2n-1} + z_{2n-1}, 2x_0 + y_0 + z_0, ...,\\
&2x_{2n-2} + y_{2n-2} + z_{2n-2}, 2x_{2n-1} + 2y_{2n-1} + z_{2n-1}, 2x_0 + 2y_0 + z_0, ...,\\
&2x_{2n-2} + 2y_{2n-2} + z_{2n-2}).
\end{aligned}$$

On the other hand, if $\Phi_2(t) = \Phi_2(t_0, t_1, ..., t_{2n-1})$

$$\begin{aligned}
&= (z_0, z_1, ..., z_{2n-1}, y_0 + z_0, y_1 + z_1, ..., y_{2n-1} + z_{2n-1}, 2y_0 + z_0, 2y_1 + z_1, ..., 2y_{2n-1} + z_{2n-1},\\
&x_0 + z_0, x_1 + z_1, ..., x_{2n-1} + z_{2n-1}, x_0 + y_0 + z_0, x_1 + y_1 + z_1, ..., x_{2n-1} + y_{2n-1} + z_{2n-1},\\
&x_0 + 2y_0 + z_0, x_1 + 2y_1 + z_1, ..., x_{2n-1} + 2y_{2n-1} + z_{2n-1}, 2x_0 + z_0, 2x_1 + z_1, ..., 2x_{2n-1} + z_{2n-1},\\
&2x_0 + y_0 + z_0, 2x_1 + y_1 + z_1, ..., 2x_{2n-1} + y_{2n-1} + z_{2n-1},\\
&2x_0 + 2y_0 + z_0, 2x_1 + 2y_1 + z_1, ..., 2x_{2n-1} + 2y_{2n-1} + z_{2n-1})
\end{aligned}$$

then we have $\sigma^{\otimes 9}.\Phi_2(t) = \sigma^{\otimes 9}(t_0, t_1, ..., t_{2n-1})$

$$\begin{aligned}
&= (z_{2n-1}, z_0, ..., z_{2n-2}, y_{2n-1} + z_{2n-1}, y_0 + z_0, ..., y_{2n-2} + z_{2n-2}, 2y_{2n-1} + z_{2n-1},\\
&2y_0 + z_0, ..., 2y_{2n-2} + z_{2n-2}, x_{2n-1} + z_{2n-1}, x_0 + z_0, ..., x_{2n-2} + z_{2n-2},\\
&x_{2n-1} + y_{2n-1} + z_{2n-1}, x_0 + y_0 + z_0, ..., x_{2n-2} + y_{2n-2} + z_{2n-2},\\
&x_{2n-1} + 2y_{2n-1} + z_{2n-1}, x_0 + 2y_0 + z_0, ..., x_{2n-2} + 2y_{2n-2} + z_{2n-2},\\
&2x_{2n-1} + z_{2n-1}, 2x_0 + z_0, ..., 2x_{2n-2} + z_{2n-2},\\
&2x_{2n-1} + y_{2n-1} + z_{2n-1}, 2x_0 + y_0 + z_0, ..., 2x_{2n-2} + y_{2n-2} + z_{2n-2},\\
&2x_{2n-1} + 2y_{2n-1} + z_{2n-1}, 2x_0 + 2y_0 + z_0, ..., 2x_{2n-2} + 2y_{2n-2} + z_{2n-2}).
\end{aligned}$$

Theorem 3.2. *A code C with length $2n$ over R_1 is a cyclic code if and only if $\Phi_2(C)$ is a quasicyclic code of index 9, with length $18n$ over $\mathbb{F}_3$.*

Proof. If C is a cyclic code , $\tau(C) = C$. Then have $\Phi_2(\tau(C)) = \Phi_2(C)$, we have $\sigma^{\otimes 9}(\Phi_2(C)) = \Phi_2(\tau(C)) = \Phi_2(C)$ from Lemma 3.2. So $\Phi_2(C)$ is quasicyclic code of index 9 . Conversely, if $\Phi_2(C)$ is quasicyclic code of index 9 , then $\sigma^{\otimes 9}(\Phi_2(C)) = \Phi_2(C)$. By using the Lemma 3.2 , we have $\sigma^{\otimes 9}(\Phi_2(C)) = \Phi_2(\tau(C)) = \Phi_2(C)$. Since Φ_2 is injective then $\tau(C) = C$.

□

Using the above theories the main conclusion is given below:

Corollary 3.1. A code C with odd lenght n over R is a cyclic code if and only if $\Phi_2(\Phi_1(C))$ is quasicyclic code of index 9 and with lenght $18n$ over $\mathbb{F}_3$.

□

4. Conclusion

It is presented the finite ring $\mathbb{F}_3 + v\mathbb{F}_3 + u\mathbb{F}_3 + u^2\mathbb{F}_3$ where $u^3 = 0$,$v^2 = 0$ and $uv = vu = 0$. It is acquired that the Gray image of a cyclic code over R with lenght n.

References

[1] Roman S., Coding and Information Theory, Graduate Texts in Mathematics. *Springer Verlag*, (1992).

[2] Wolfmann J., Negacyclic and cyclic codes over $Z4$. *IEEE Trans. Inf. Theory*, 45, (1999), 2527-2532.

[3] Ling S., Blackford J. T., $\mathrm{Z}_{p^{k+1}}$_linear codes. *IEEE Trans. On Inform. Theory*, 45, (2006), no.9, 2592-2605.

[4] Jian-Fa,Q.,Zhang L.N.,Zhu S.X.,$(1+u)$- cyclic and cyclic codes over the ring $\mathbb{F}_2 + u\mathbb{F}_2$. *Applied Mathematics Letters*,19, (2006), 820-823.

[5] Jian-Fa,Q.,Zhang L.N.,Zhu S.X., Constacyclic and cyclic codes over $\mathbb{F}_2 + u\mathbb{F}_2 + u^2\mathbb{F}_2$. *IEICE Trans. Fundamentals*, 89, (2006), no.6, 1863-1865.

[6] Amarra M.C., Nemenzo F.R,On $(1-u)-$cyclic codes over $\mathbb{F}_p^k + u\mathbb{F}_p^k$.*Applied Mathematics Letters*, 21, (2008), 1129-1133.

[7] Udomkavanich P., Jitman S. ,On the Gray image of $(1-u^m)-$cyclic codes $\mathbb{F}_{p^k} + u\mathbb{F}_{p^k} + \ldots + u^m\mathbb{F}_{p^k}$.*Int.J.Contemp. Math. Sciences*, 4, (2009), no.26, 1265-1272.

[8] S.Karadeniz, B.Yildiz,On $(1+v)-$constacyclic codes over $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2 + uv\mathbb{F}_2$.*Journal of the Franklin Institute*, 348, (2011), 2625-2632.

[9] B.Yildiz,S.Karadeniz,Linear codes over $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2 + uv\mathbb{F}_2$.*Des. Codes Cryptgr.*, 54, (2010), 61-71.

[10] Xu Xioafang,$(1+v)-$*constacyclic codes over* $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2$. *Computer Engineering and Applications*,49 , (2013), no.12, 77-79.

Affiliations

MUSTAFA ÖZKAN
ADDRESS: Trakya University, Dept. of Mathematics, 22030, Edirne-Turkey.
E-MAIL: mustafaozkan@trakya.edu.tr

FIGEN ÖKE
ADDRESS: Trakya University, Dept. of Mathematics, 22030, Edirne-Turkey.
E-MAIL: figenoke@gmail.com