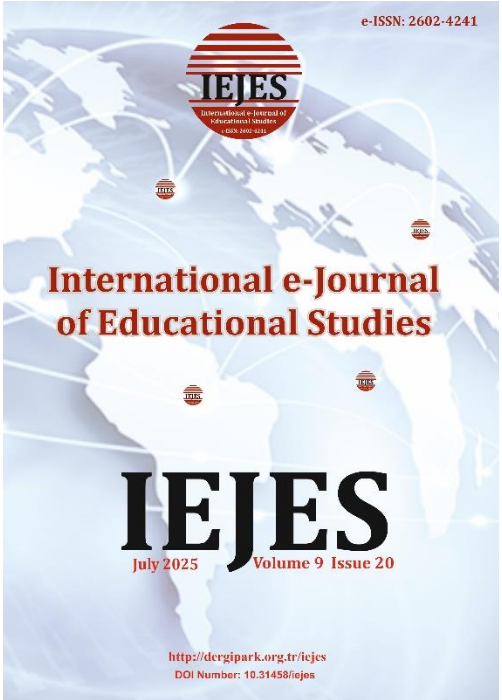

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Abstract

This study examines the strategies used by middle school students in numerosity estimation and the level of closeness of their estimations to the actual value. A special case study method was used, and 71 students from fifth to eighth grades in a middle school in the Central Black Sea Region participated in the study. The data were analysed using descriptive and content analysis methods. During the analysis of the data, MAXQDA, a software program designed for computer-aided qualitative and mixed exchange data, text and multimedia analysis, was utilized. The findings were presented in the form of MAXQDA maps and quotations from the participants' answers were included. It was concluded that the students mostly made random estimations; that is, they did not use any strategies. Accordingly, it was revealed that they could not estimate close to the actual value, that is, their numerosity estimation skills were low. The most used strategy, other than random estimation, was benchmark comparison, and this strategy provided estimates closer to the actual value. It was also observed that the strategies changed depending on the situation to be estimated.

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Research Article

Examining the Numerosity Estimation Strategies and Skills Used by Middle School Students*

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Abstract

This study examines the strategies used by middle school students in numerosity estimation and the level of closeness of their estimations to the actual value. A special case study method was used, and 71 students from fifth to eighth grades in a middle school in the Central Black Sea Region participated in the study. The data were analysed using descriptive and content analysis methods. During the analysis of the data, MAXQDA, a software program designed for computer-aided qualitative and mixed exchange data, text and multimedia analysis, was utilized. The findings were presented in the form of MAXQDA maps and quotations from the participants' answers were included. It was concluded that the students mostly made random estimations; that is, they did not use any strategies. Accordingly, it was revealed that they could not estimate close to the actual value, that is, their numerosity estimation skills were low. The most used strategy, other than random estimation, was benchmark comparison, and this strategy provided estimates closer to the actual value. It was also observed that the strategies changed depending on the situation to be estimated.

Keywords: Numerosity estimation, estimation strategies, estimation skills, middle school students

1. INTRODUCTION

Estimation skills are one of the basic skills required for adaptation to daily life, as they enable mathematical operations to be performed without a pen or paper (Baran-Kaya & Çelik-Demirci, 2022), and they are common activities that everyone uses more or less in their lives (Andrews, Xenofontos & Sayers, 2021). As it is known, a definite answer is not required, and an approximate answer is sufficient in all situations and tasks that require estimation (Sowder & Wheeler, 1989). If we consider examples such as "How long does it take to finish housework?", "How much does a large pizza cost?", "How many people were at the match?", or "How much is 75 × 31 approximately?", it is possible to say that estimation skills are used more frequently in daily life than all other quantitative determination processes (Siegler & Booth, 2005). Estimation is also an essential part of mathematical understanding (Siegler & Booth, 2005) because it is related to specific aspects of mathematical ability, e.g. arithmetic ability and general measures of mathematical ability, e.g. achievement scores (Dowker, 2003). Moreover, proportional reasoning, problem-solving, and estimation skills are the three key mathematical thinking skills from the perspective of preschool, primary and middle school teachers (Sriraman & Knott, 2009). In addition, estimation activities encourage students to connect the mathematical concepts they are learning and the skills they are developing (The National Council of

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Teachers of Mathematics [NCTM], 2000). Another basis for the importance of estimation is that it provides practicality because it requires immediately recognizing the size or quantity of something without counting or measuring it (Micklo, 1999). However, school-age children and even many adults are far from being good at estimation (Siegler & Booth, 2005).

There are various definitions of estimation and estimation skills in the literature. Reys (1986) defines estimation as the process of providing an adequate answer to a problem, Siegler and Booth (2005) as the process of translating between alternative quantitative representations, at least one of which is uncertain, Micklo (1999) as generating ideas quickly and logically without requiring counting or measuring, Aytekin (2012) as the art of determining the closest to the actual value, and Bağdat and Yıldız (2023) as finding the closest value to the actual value using intuition or reasoning skills. There are also various classifications of estimation skills with various definitions in the literature. While Segovia and Castro (2009) examined estimation skills under two subheadings as metric and operational estimation, Hogan and Brezinski (2003), Munakata (2002), and Newcomb (2014) addressed estimation skills under three headings as operational estimation, metric estimation, and numerosity estimation. Sayers, et al. (2020) also stated that there is number line estimation in addition to these three types of estimations. Numerosity estimation is considered in this study.

Numerosity estimation is the ability of a person to estimate the number of people or objects in a given location without physically counting the items (Newcomb, 2014). When counting one by one is not possible, the number of items can be estimated to provide an approximate assessment of their number (Anobile et al., 2020; Hogan & Brezinski, 2003). Numerosity estimation is different from other types. If estimation is considered as a process of translating between quantitative representations, as defined by Siegler and Booth (2005), operational estimation involves translating from one numerical representation (e.g., 75 × 29) to another numerical representation (approximately 2,200). However, numerosity estimation requires translating a non-numerical quantitative representation (e.g., a visual representation of the approximate volume and density of candies in a jar) into a number (Brumm & Rathgeb-Schnierer, 2023; Siegler & Booth, 2005).

In addition to simple arithmetic operations such as addition or multiplication, various strategies such as decomposing and recomposing the quantity to be estimated must be used to make a numerosity estimation (Crites, 1992; Luwel & Verschaffel, 2008; Siegel et al., 1982). The strategies used for numerosity estimation vary depending on the situation in which the quantity is to be estimated in the literature (Siegler & Booth, 2005). Crites (1992) applied a test with estimation questions to third, fifth and seventh grade primary school students in his study and determined the numerosity estimation strategies used by the students. Crities (1992) stated that there are three basic strategies: basic measurement comparison, decomposition/rearrangement and eyeball estimation. Another strategy is comparing a known quantity by determining the reference point used to estimate an unknown quantity. An example of this strategy is "We know that 600 beans fit in a jar, a bean is about one-third the size of a popcorn kernel, so about 200 kernels can fit in the same jar" (Baran-Kaya & Çelik-Demirci, 2022; Crites, 1992; Siegel et al., 1982). In addition to these strategies, the estimation strategy based on knowledge and experience is one of the strategies used when making a numerosity estimation (Baran- Kaya & Çelik-Demirci, 2022; Tekinkır, 2008).

Studies on numerosity estimation are limited numerically compared to other types (Kayhan-Altay et al., 2023). Some studies have determined the various strategies when making numerosity estimations (Baran-Kaya & Çelik-Demirci, 2022; Crites, 1992; Lang, 2001; Siegel et al., 1982). Some studies have also compared the numerosity estimation performances of students with their mathematical performances. While the majority of these studies have put forward that students with better estimations are more successful in standardized mathematics tests and other mathematical abilities and understanding (Ashcraft & Moore, 2012; Booth & Siegler, 2008; Dowker, 2003; Schneider et al., 2008; Siegler & Booth, 2004), Brumm, and Rathgeb-Schnierer (2023) have suggested that there is no relationship between success in numerosity estimation and interest in mathematics and mathematical achievement. The number of studies on estimation is limited in the domestic literature (Boyraz & Akgün, 2017), and they are usually studies in the types of operational and measurement estimation. In addition, although operational and measurement estimation skills and strategies have been included in the curriculum since 1948 (Bulut et al., 2017), it is also striking that numerosity estimation is not directly included. Moreover, the number of studies directly addressing numerosity estimation in the domestic literature is also limited. Among these studies, Kayhan-Altay et al. (2023) examined numerosity estimation performances and strategies of preschool students. They concluded that students performed better when a reference point was presented compared to open-ended estimation situations. It was observed that some studies included this type of estimation, although they did not directly address numerosity estimation. Baran-Kaya and Çelik-Demirci (2022) examined the numerosity estimation strategies of prospective mathematics teachers, along with other types of estimation, and the closeness of the estimations made using these strategies to the actual value. It was revealed that the estimations of participants who used the basic measurement comparison strategy while making numerosity estimations were better. Ev-Çimen and Bilgiç (2022), on the other hand, revealed that there was the least content regarding numerosity estimation in their study where they addressed the types of estimation in textbooks. In addition, the fact that strategies related to numerosity or other types of estimation are not included in textbooks and that students are not taught basic estimation strategies (Tekinkır, 2008) is a situation that should be considered. However, as it is known, estimation skills are a fundamental part of learning mathematics (Brumm & Rathgeb-Schnierer, 2023), and it requires going beyond the memorization of procedures and applying mathematical knowledge in flexible ways (Siegler & Booth, 2005). Considering that numerosity estimation is also related to the ability to perform simple arithmetic operations (Brumm & Rathgeb-Schnierer, 2023), arithmetic success (Wong et al., 2016), and mathematics success (Booth & Siegler, 2006), it can be considered a type of estimation that should be emphasized.

When we examine the related studies, it is clear that the number of studies conducted directly on the numerosity estimation skill strategies and the closeness of these strategies to reality is quite limited. Accordingly, although this study only targets numerosity estimation, it includes a variety of numerosities where the objects can be counted, whether they are limited or not, presented regularly or irregularly, and where various estimation strategies can be used. This study aims to examine the strategies preferred by middle school students while making numerosity estimations and the degree of closeness of their estimations to reality.

2. METHOD

The case study method allows the situation to be investigated in detail and to reveal how the factors related to the situation affect the situation or how it is affected by the situation (Yıldırım & Şimşek, 2021). This study used the case study method since it aimed to examine in detail the numerosity estimation strategies and estimation skills used by middle school students.

2.1. Study Group

The study group consists of 71 students who attend fifth, sixth and seventh grades in the 2022-2023 academic year at a middle school in the Central Black Sea Region. Nineteen students are in fifth-grade, twenty six are in sixth-grade, and twentysix are in seventh-grade. Since there were no volunteer students from the eighth grade students, 8th grade students could not be included in the study. The study group was determined using the easily accessible sampling method, one of the purposeful sampling types. The principle of volunteering was considered during the collection of data, and code names (S1, S2, ...) were used for the students to ensure confidentiality and anonymity.

2.2. Data Collection Tool

The "Numerosity Estimation Skill Test (NEST)" with six questions prepared by the researchers for middle school students was used as a data collection tool in this study. In the first stage, two experts who studied estimation skills gave their opinions on this test with ten questions developed for middle school students. Later, NEST was applied as a pilot study to twenty students not included in the study group, and they gave their opinions regarding the questions. Some questions were removed, and some were subject to contextual changes according to the opinions and the answers. Questions were from daily life as much as possible. In addition, assuming that various estimation strategies could be used, care was taken to use visuals with both a small number of objects and a large number of objects and where these objects were presented regularly or irregularly. Students were asked not only to provide their estimations for each question but also to explain the reasoning behind them. It was aimed to determine the students' estimation strategies in this way. Figure 1 shows a sample question in NEST.

Question 2:

In the image on the left, there are students performing a demonstration in a gymnasium. How many students are approximately involved in this demonstration? Please explain how you arrived at your estimate.

Your estimate	
The explanation	

Figure 1. A question in NEST

2.3. Analysis of Data

The descriptive analysis technique was used for the analysis, but content analysis was also performed from time to time. Before analysis, a draft coding template was created. This template includes two dimensions: numerosity estimation strategies and estimation skills based on the closeness and distance of the estimations to the actual value. According to the template, one of the researchers performed the coding using the MAXQDA qualitative data analysis program. After coding, another researcher randomly selected ten data and coded them independently of the other coder. These codes were then compared, and the few inconsistencies were resolved.

The MAXQDA qualitative data analysis program was used during the analysis. First, the strategies were determined by considering the numerosity estimation strategies in the literature (Crities, 1992; Lang, 2001; Siegel et al., 1982), and then the closeness of the strategies to actual value was evaluated as "fairly close", "close", and "far". There are various criteria in the literature regarding the grading of closeness to actual value (Baroody & Gatzke, 1991; Crites, 1992; Gatze, 1989; Siegel et al., 1982). The criteria in Levine's (1982) study were considered in this study. Accordingly, if the closeness of the answer to the actual value was between 0-10%, it was evaluated as "fairly close", if between 10%-30%, as "close", and if it was more than 30%, as "far". A sample student (S32) answer and the coding of this answer are given below:

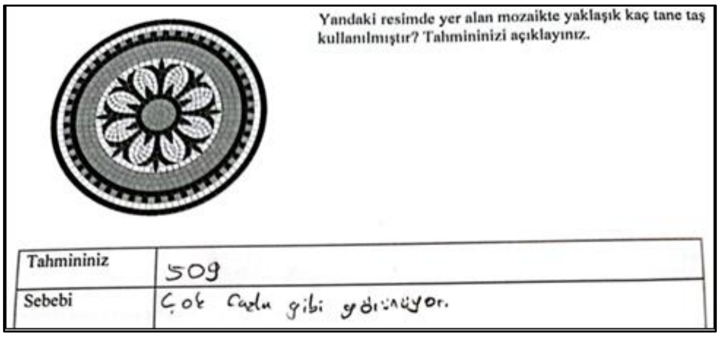

Figure 2. **A sample student answer and the coding of this answer**

Since S32 did not base his estimate on a reason, his answer was coded as a random estimation strategy. In addition, since the number of stones in the mosaic was 1058, the student's answer was evaluated as far from the actual value. The strategies used in the questions and the closeness to actual value were analysed separately. Afterwards, all the questions were evaluated, and the percentage of strategies being preferred and reaching the actual value were calculated. The MAXQDA maps as a result of the analysis and quotes from the participant answers were included in the findings.

3. FINDINGS

The findings are presented in order according to the questions in the data collection tool. 71 students had a total of 426 answers in NEST. Table 1 shows the strategies used by the students in making numerosity estimations and the frequencies of the closeness of these strategies to the actual value.

Table 1. Students' numerosity estimation strategies and their closeness to the actual value

Strategy	Frequency of Use	Closeness to the actual value		
		Fairly close	Close	Far
Random	157	27	29	101
Benchmark comparison	90	34	32	34
Comparison	55	3	2	50
Counting	54	21	15	18
Based on knowledge/experience	24	2	11	11
Eyeball estimation	19	2	7	10
No answer	17	-	-	-
No explanation	10	2	1	7

According to Table 1, middle school students usually estimate randomly; that is, they do not use any strategies. These random estimations are far from the actual value. The most commonly used strategy after random estimation is benchmark comparison. The majority of the estimations using this strategy are close to the actual value. The majority of the participants who used the strategy of making comparisons using a reference point estimated quite far from the actual value. Some middle school students avoided estimating and made direct counting. It is striking that the few estimations close to the actual value were not enough despite the direct counting. The percentages of using all strategies and reaching the actual value are given below in detail for each problem situation.

Students were asked to estimate the number of seats by giving a visual of a conference hall in the first question. The actual number of value was 270 seats. Students' estimates were evaluated as "fairly close" if they were between 243 and 297, "close" if they were between 189 and 242 and 298 and 351, and "far" if they were outside these ranges. Figure 3 shows the MAXQDA map containing the findings from the first question.

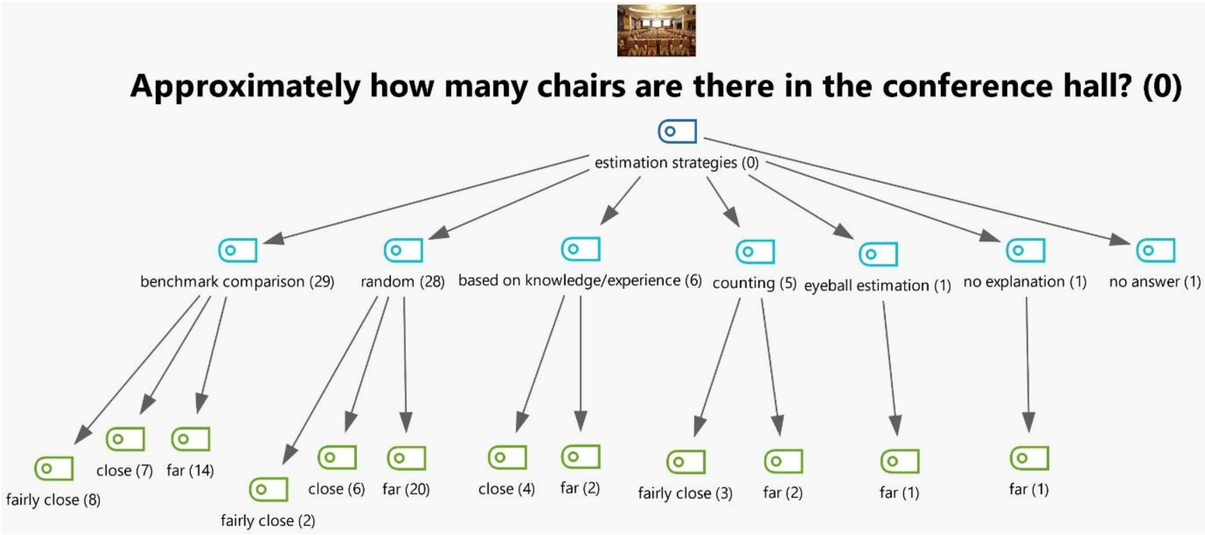

Figure 3. Findings related to the first question

Figure 3 shows that 42.25% of estimates are fairly close or close to the actual value. 57.75% of the students estimated far from the actual value. Most of the students (f=29) used the benchmark comparison strategy. A student who used this strategy said: "*First, I counted the tables and then the chairs, and finally I multiplied the number of chairs and the tables. 13x12= 336*" and estimated close to the actual value. S36, who used the same strategy, estimated far from the actual value and said: "*Since there are six chairs in each row, 6x3=18 18x10=180*". Another most used strategy in this question is random estimation (f=28). 71.42% of those who used random strategy made estimates far from the actual value. S8, one of the students who estimated that was far from the actual value, said: "*Because the conference hall is big and I think there are probably 100 to 200 seats, that's why I wrote 175.*" 224

In the second question regarding the numerosity estimation, students estimated the approximate number of students by giving a visual of students performing in the gym. The actual value was 110. The students' estimates were evaluated as "fairly close" if they were between 99 and 121, "close" if they were between 77 and 98 and 122 and 143, and "far" if they were outside these ranges. Figure 4 shows the MAXQDA map containing the findings obtained from this question.

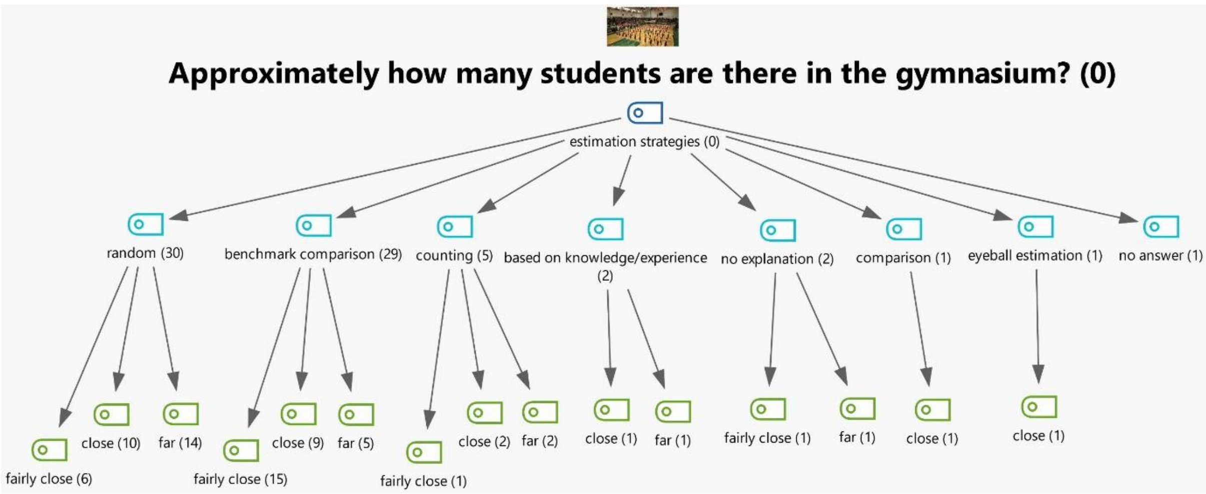

Figure 4. Findings related to the second question

Figure 4 shows that 33.8% of the estimates are close, 32.39% are far, and 32.39% are fairly close to the actual value. The students usually used random estimations (f=30) and benchmark comparison strategies (f=29). Five students did not estimate and answered by directly counting. S23, who estimated far from the actual value by using the benchmark comparison strategy, said: "*15 students were lined up, and I saw that there were 11 students in that group, so 15x11=165*". S3, who did not use any strategy and estimated randomly, said: "*50 because there were many children*", and it was far from the actual value. S56, who answered close to the actual value, said: "*I compared it with my friends who I played with in the park*" and estimated based on his experiences.

Students were given a visual of a library and asked to estimate the number of books in the third question. There are 132 books in the library. The students' estimations were evaluated as "fairly close" if the answers were between 118 and 146, "close" if they were between 92 and 117 and 147 and 172, and "far" if they were outside these ranges. Figure 5 shows the MAXQDA map containing the findings from question 3.

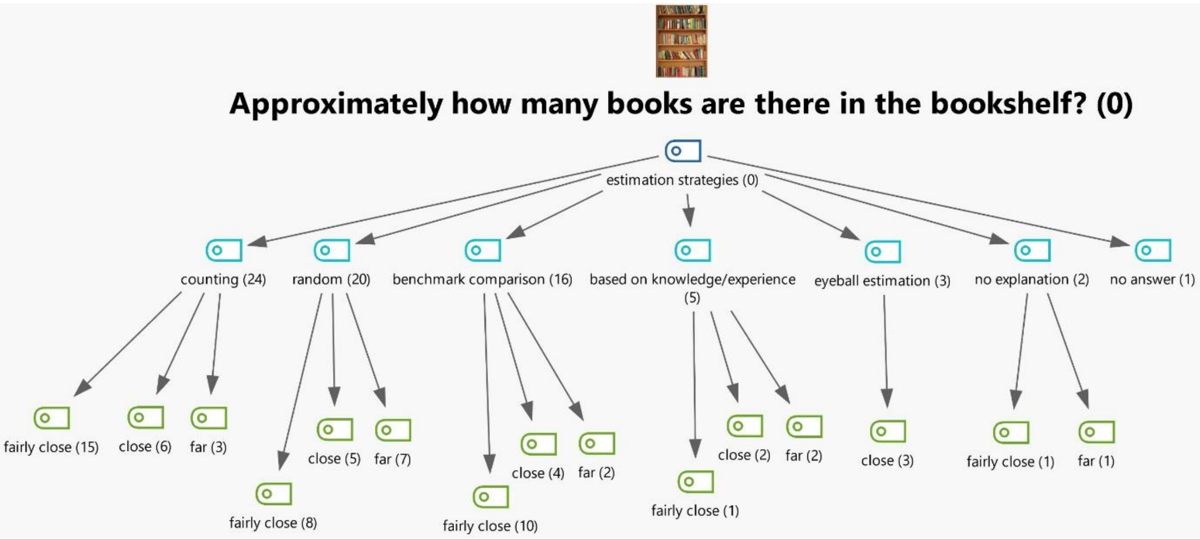

Figure 5. Findings related to the third question

Figure 5 shows that 49.29% of the answers are fairly close, 28.16% are close, and 21.12% are far from the actual value. It is striking that the majority of the students made exact counting (f=24) without estimating and estimated randomly (f=20). The students estimated the closest estimation to the actual value other than counting (f=10) using the benchmark comparison strategy. One of these students, S54, estimated fairly close to the actual value and said: "*I found a row of 24. Since there were five shelves, I multiplied it by 5 and added 3 extra books*". Three students who used the eyeball strategy also estimated close to the actual value. S2, who estimated fairly close to the actual value, stated that he counted without using any strategy and said: "*counting the books*". Some participants answered close to the actual value (f=6) and far from it (f=3) despite counting like S2. S82 found a close answer to the actual value by using a strategy based on his experiences with the explanation, "*It is 115 because I compared it with the school library and saw that there was not much difference.*"

Students were given a visual of a hospital parking lot and asked to estimate the number of vehicles in the fourth question. There were exactly 263 vehicles. Accordingly, the estimations were evaluated as "fairly close" if they were between 236 and 289, as "close" if they were between 184 and 235 and 290 and 341, and as "far" if they were outside these ranges. Figure 6 shows the MAXQDA map containing the findings from the fourth question.

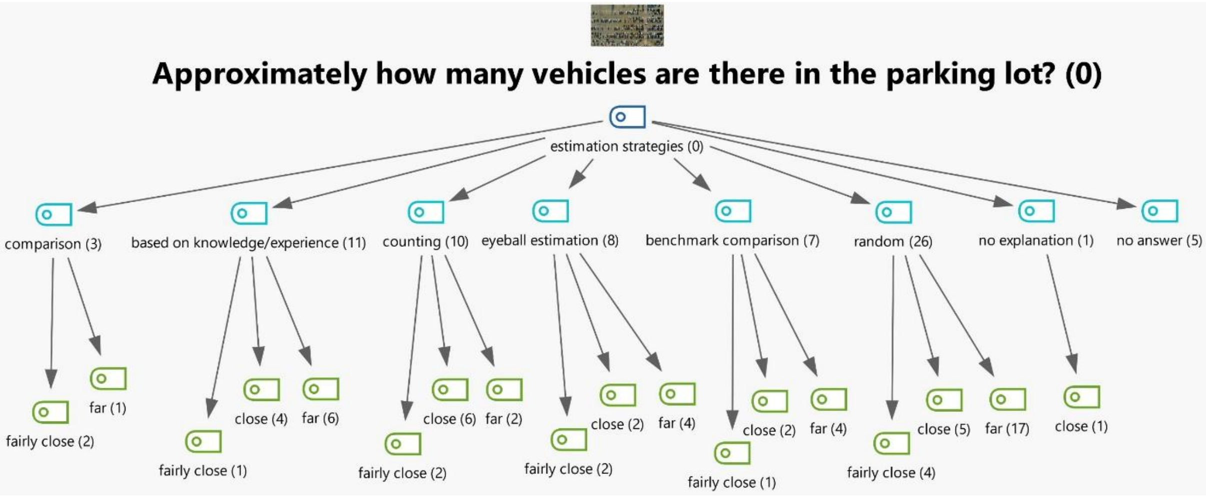

Figure 6. Findings related to the fourth question

Figure 6 shows that almost half of the students (47.88%) estimated far from the actual value, 28.16% close to the actual value, and 16.9% fairly close to the actual value. 7.04% of the students did not answer this question. It was observed that the students mostly did not use an estimation strategy in this question and made a random estimation (f=26). In addition, the strategy with the highest number of estimations far from the actual value was again the random estimation strategy. S40 answered 500 by saying "I think so" and made a random estimation very far from the actual value. S33, one of the students who used the strategy of relying on knowledge/experience, estimated far from the actual value and said: "*My mother was working at the hospital, I sometimes went down to the garden and counted cars because I was bored. I wrote down twice the number I counted. 1251+1251=2502*". Two of the three students who used the comparison strategy also estimated fairly close to the actual value. S64, who used the comparison strategy, said: "*I compared it to the garage of our building.*" It is striking that 10 students answered by counting without using any strategy, yet the majority of them estimated far from the actual value.

In another question about the numerosity estimation, students were given a mosaic image and asked to estimate how many stones it consisted of. The actual value was 1058. Estimations were evaluated as "fairly close" if they were between 952 and 1163, as "close" if they were between 740 - 951 and 1164 - 1375, and as "far" if they were outside these ranges. Figure 7 shows the MAXQDA map containing the findings obtained from this question.

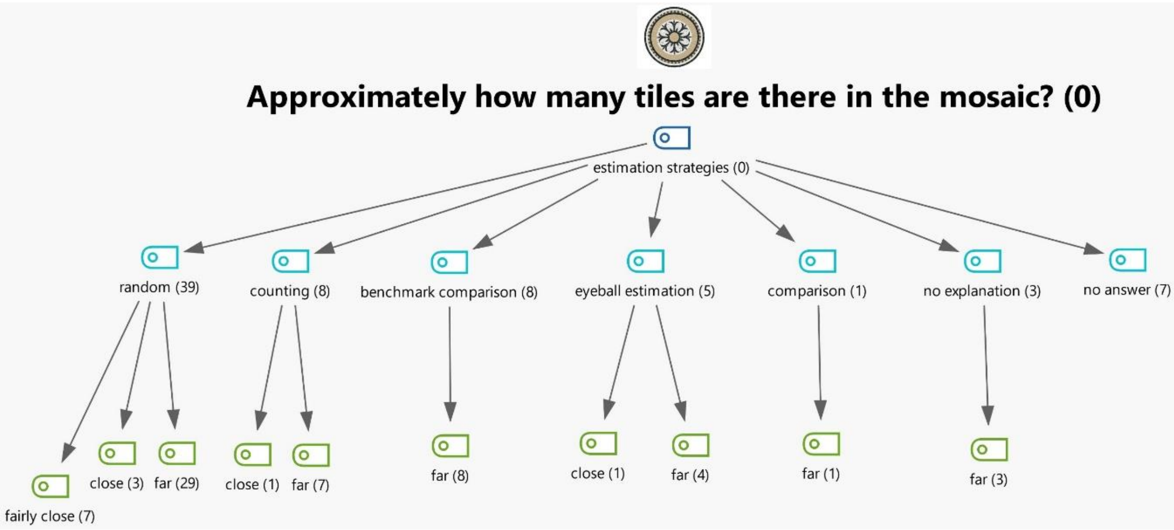

Figure 7. Findings related to the fifth question

Figure 7 shows that 73.23% of the answers are far from the actual value, 10% are fairly close, and 9.85% are close. 9.85% did not answer the question. Most of the students used the random estimation (f=39) strategy, and only seven students estimated fairly close to the actual value. Among the students who estimated randomly, S64 said: "*I don't think it is 200 or so. It is 100 at most.*" and estimated that is very far from the actual value. S18, who answered close to the actual value, said: "*It is 1000 because they take up a lot of space because they are small stones.*" Although he estimated randomly, it is very close to the actual value. All of the students who used the benchmark comparison strategy estimated far from the actual value. S5 estimated far from the actual value and said: "*There were 104 in Part 1, I counted them and thought that each side had 10 stones less and thought that there were 170 stones in the middle flower and added them all up*." He proceeded with the right strategy, but since he underestimated the number of stones in the middle flower motif, he estimated far from the actual value. Although the number of stones was quite high, almost all of the students who counted directly instead of estimating answered far from the actual value. For example, S54 said: "I counted the stones" and gave the answer 209.

In the last question regarding the numerosity estimation, there was a visual of two jars, one filled with olives and the other with chickpeas. There were 84 olives in the jar full of olives, and the students were asked to estimate how many chickpeas might be in the other jar. The actual value was 572 in this question. The values between 514 and 629 were evaluated as "fairly close", between 400 and 513 and 630 and 744 as "close", and outside these ranges as "far". Figure 8 shows the MAXQDA map containing the findings from the sixth question.

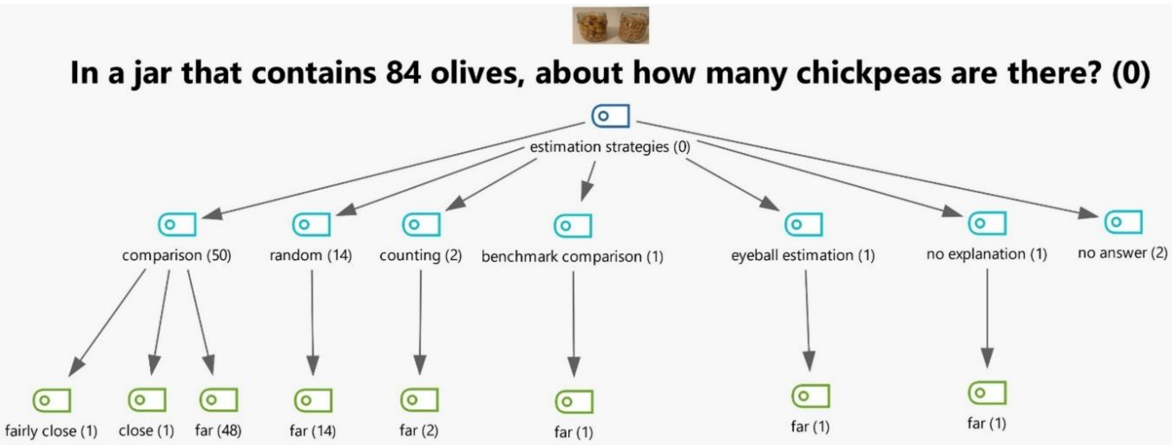

Figure 8. Findings related to the sixth question

Figure 8 shows that 94.36% of the answers are far from the actual value, 1.4% are fairly close, and 1.4% are close. 2.81% did not answer this question. Most of the students used the comparison (f=50) strategy by taking a reference point. However, only one student could estimate fairly close, and one student could estimate close to the actual value. S28 estimated closely and said: "*Because the olives are bigger but the chickpeas are much smaller, so there is a difference between them. It is 500*". S43 estimated closely: "*Because the olives in the first jar are big, the chickpeas are small, so it takes up less space, so it is more than 84, it is 140*". However, it is seen that he estimated far from the actual value because he could not compare the sizes of the olives and chickpeas correctly. S56 followed the correct strategy by saying: "*If each olive has 3 chickpeas, 84 + 84 + 84 = 252*", but he could not estimate closely because he did not think that chickpeas could fit into the spaces created by the olives in the jar.

4. DISCUSSION and CONCLUSION

The majority of the students' estimations were quite far from the actual value in this study which examined the strategies of middle school students regarding numerosity estimation and the closeness of their estimations to the actual value. In addition, some participants didn't answer the questions or answered but did not explain the reason. Considering all these, middle school students' numerosity estimation skills are low. Baran-Kaya and Çelik-Demirci (2022), in their study with prospective teachers, revealed that the majority of the participants gave answers close to the actual value, especially in questions containing visuals, when making numerosity estimations. However, this difference may be due to the development of numerosity estimation performance and strategy use with increasing age and experience (Brumm & Rathgeb-Schnierer, 2022; Luwel et al., 2005). Similarly, Siegel et al. (1982) stated that 6th-8th-grade students made worse mass estimations than adults.

It can be concluded that the reason why most of the middle school students' numerosity estimates are far from the actual value is due to the use of a random estimation strategy. Moreover, other studies have also shown that making a random estimate causes incorrect estimates (Baran-Kaya & Çelik-Demirci, 2022) because, although it is known that strategy choices in numerosity estimation depend on certain problem characteristics (Crites, 1992; Siegler & Booth, 2005), it is a fact that certain types of strategies are more effective in numerosity estimation (Luwel et al., 2005). One of these strategies is undoubtedly the benchmark comparison strategy (Baran-Kaya & Çelik-Demirci, 2022). The most used strategy after random estimation was the benchmark comparison strategy among the participants who were 5th-8th-grade middle school students who made both close and far estimations. In addition, most of the students who preferred the benchmark comparison strategy were able to estimate close to the actual value. Similarly, the most preferred strategy of the 5th-7th-grade students in the study of Crites (1992) and the middle school prospective mathematics teachers in the study of Baran-Kaya and Çelik-Demirci (2022) is the benchmark comparison.

Although some strategies are preferred more, estimation strategies can vary depending on the situation or problem (Luwel et al., 2003; Siegler & Booth, 2005). According to this study various strategies were preferred more by middle school students in some numerosity estimation questions. When the number of olives in a jar was given, and the number of chickpeas was asked, students used the comparison strategy more. They tried to estimate by comparing the size of the olives in the visual with the size of the chickpeas. However, the majority of the estimations were far from the actual value since they could not compare the sizes correctly and could not think that the gaps in the jar containing the olives would also be filled by chickpeas. Similarly, if the situation that the students were asked to estimate consisted of a smaller number of objects, such as the number of books on a bookshelf, they preferred to count the objects one by one instead of estimating. It is striking that students answered far from the actual value despite counting one by one. When the number of objects to be estimated is high, instead of implementing a strategy, they estimate randomly, which means that they avoid using a strategy for estimation. In addition, it has been understood that their estimates are far from the actual value due to the effect of estimating randomly when the number of objects is high. Russo et al. (2022) also revealed that both second and sixth-grade students have difficulty in estimating close to the actual value when the number of objects increases. It was observed in the study by Kayhan-Altay et al. (2023) that the most preferred strategy in numerosity estimation is random answers such as "it seems to me" without making any comments about the quantities.

In addition to the frequently used strategies mentioned above, some participants use the eyeball strategy (Crites, 1992), a perceptual-based strategy in which the elements are visually scanned, and the strategy of estimating based on knowledge/experience. Although there are a limited number of studies in which the strategy of estimating based on knowledge/experience is used in numerosity estimation (Baran-Kaya & Çelik-Demirci, 2022), Gandini et al. (2008) stated that strategies that do not correspond to other strategies in the literature can also be used. It can be thought that the use of questions that can encourage the use of various strategies in the data collection tool may have led to the emergence of various types of strategies.

While all other questions require estimating in more regular structures in the NEST used as a data collection tool in the study, there is a more irregular structure in the case where the number of stones in the mosaic needs to be estimated. Moreover, this was the situation where students had the most difficulty when making a numerosity estimation. While many students didn't answer the question, lots of them who answered estimated far from the actual value. This result is consistent with the studies by Smith (1999) and Russo, et al. (2022).

When the results are examined in general, the failure of middle school students in numerosity estimation may be because numerosity estimation is rarely mentioned in the curriculum and textbooks (Ev-Çimen & Bilgiç, 2022). Moreover, students can only associate mathematics with the world of exact numbers unless otherwise shown (Brumm & Rathgeb-Schnierer, 2022), and estimation can be seen for them as a contrast to the world of exact numbers (Schipper, 2009). For this reason, numerosity estimation skills and strategies can be given more importance in both mathematics curriculum and teaching practices starting from the younger grades because the earlier children experience estimation, the more effective they can estimate in the later period (Micklo, 1999). Therefore, teachers can draw attention to their students' use of numerosity estimation strategies in terms of counting that students may indirectly encounter in many mathematics subjects.

This study evaluated numerosity estimation skills of fifth and seventh-grade middle school students from a holistic perspective. In another study, numerosity estimation skills at various grade levels can be comparatively addressed in other studies since it is known that age and education level have an effect on estimation skills.

Ethics Committee Decision

This research was carried out with the permission of Kırıkkale University Publication Ethics Board with the decision numbered 246751 dated 22.04.2024.

5. REFERENCES

Andrews, P., Xenofontos, C., & Sayers, J. (2021). Estimation in the primary mathematics curricula of the United Kingdom: Ambivalent expectations of an essential competence, *International Journal of Mathematical Education in Science and Technology*. *53*(8), 2199–2225. https://doi.org/10.1080/0020739X.2020.1868591

Anobile, G., Castaldi, E., Moscoso, P. A. M., Burr, D. C., & Arrighi, R. (2020). “Groupitizing”: a strategy for numerosity estimation. *Scientific Reports*, *10*(1), 13436.

Ashcraft, M. H., & Moore, A. M. (2012). Cognitive processes of numerical estimation in children. *Journal of Experimental Child Psychology*, *111*(2), 246-267.

Aytekin, C. (2012). *İlköğretim ikinci kademe öğrencilerinin kesirlerde tahmin becerilerinin incelenmesi [Investigating the middle school students' estimation ability with respect to fractions]*. Master's thesis, Abant İzzet Baysal University, Bolu. Retrieved from

Bağdat, A. & Yıldız, B. (2023). Türkiye'de 2012-2022 yılları arasında matematik eğitiminde tahmin konusunda yapılmış çalışmaların incelenmesi [An examination of studies conducted on prediction in mathematics education in Turkey between 2012-2022]. *Ulusal Eğitim Dergisi*, *3*(2), 341-360.

Baran Kaya, T., & Çelik Demirci, S. (2022). Examination of the estimation skills and strategies of pre-service elementary mathematics teachers. *International Journal of Research in Education and Science (IJRES)*, *8*(2), 243-261. https://doi.org/10.46328/ijres.2897

Baroody, A. J., & Gatzke, M. R. (1991). The estimation of set size by potentially gifted kindergarten-age children. *Journal for Research in Mathematics Education*, *22*(1), 59-68.

Booth, J. L., & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*(4), 1016-1031.

Boyraz, D. S. & Aygün, M. (2017). Türkiye'de matematikte tahmin konusuyla ilgili yapılmış çalışmalar [Studies on the subject of estimation in mathematics in Turkey]. *Milli Eğitim Dergisi*, *216*, 165-185.

Brumm, L., & Rathgeb-Schnierer, E. (2023). The relationship between accuracy in numerosity estimation, math achievement, and math interest in primary school students. *Frontiers in Psychology*, *14*, 1146458.

Bulut, S., Yavuz, F. D., & Yaman, B. (2017). Tahmin becerilerinin 1948'den 2015'e 1-5. sınıflar matematik dersi öğretim programlarındaki yeri [Estimation skill in 1-5th grades mathematics education curricula from 1948 to 2015]. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi*, *18*(1), 19-39.

Crites, T. (1992). Skilled and less skilled estimators' stratejies for estimating discrete quantities. *The Elemantary School Journal*, *92*(5), 601-620.

Dowker, A. (2003). Young children's estimates for addition: The zone of partial knowledge and understanding. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 234–265). Erlbaum

Ev-Çimen, E., & Bilgiç, E. (2022). Ortaokul matematik ders kitaplarının tahmin becerisi bakımından incelenmesi [Analysis of secondary school mathematics textbooks in terms of estimation skills]. *Uluslararası Türk Eğitim Bilimleri Dergisi*, *2022*(19), 475-500.

Gandini, D., Lemaire, P., & Dufau, S. (2008). Older and younger adults' strategies in approximate quantification. *Acta Psychologica*, *129*(1), 175-189.

Gatzke, M. R. (1989). *Kindergarten children's estimates of numerosity*. University of Illinois at Urbana-Champaign.

Hogan, T. P., & Brezinski, K. L. (2003). Quantitative estimation: One, two, or three abilities?. *Mathematical Thinking and Learning*, *5*(4), 259-280.

Kayhan Altay, M., Alkaş Ulusoy, Ç., Özer, A., & Umay, A. (2023). Examining kindergarten children's numerosity estimation skills. *Early Childhood Education Journal*, *52*(3), 503-513.

Lang, F. K. (2001). What is a "good guess" anyway? Estimation in early childhood. *Teaching Children Mathematics*, *7*(8), 462–466.

Levine, D. R. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education*, *13*(5), 350–359. https://doi.org/10.2307/749010

Luwel, K., Verschaffel, L., Onghena, P., & De Corte, E. (2003). Strategic aspects of numerosity judgment: The effect of task characteristics. *Experimental Psychology*, *50*(1), 63-75.

Luwel, K., Lemaire, P., & Verschaffel, L. (2005). Children's strategies in numerosity judgment. *Cognitive Development*, *20*(3), 448-471.

Luwel, K., and Verschaffel, L. (2008). Estimation of 'real' numerosities in elementary school children. *European journal of psychology of education*, *23*(3), 319–338.

Micklo, S. J. (1999), Estimation; Its more than a guess, *Childhood Education*, *5*(3), 142-145.

Munakata, M. (2002). *Relationships among* estimation ability, attitude toward estimation, category width and gender in students of grades 5–11*. Columbia University.

Newcomb, D. L. (2014). Mathematical estimation and its real-world application in the construction fields (Doctoral dissertation, State University of New York, New York, USA).

Reys, B. J. (1986). Estimation and mental computation ıt's "about" time. *The Arithmetic Teacher*, *34*(1), 22-23.

Russo, J., MacDonald, A., & Russo, T. (2022). The influence of making predictions on the accuracy of numerosity estimates in elementary-aged children. *International Journal of Science and Mathematics Education*, *20*(3), 531-551. https://doi.org/10.1007/s10763-021-10156-3.

Sayers, J., Petersson, R., Rosenqvist, E., & Andrews, P. (2020). Estimation: an inadequately operationalised national curriculum competence. In *Proceedings of the British Society for Research into Learning Mathematics*, *40(*1). BSRLM.

Schipper, W. (2009). *Handbuch für den mathematikunterricht an grundschulen*. Schroedel.

Schneider, M., Heine, A., Thaler, V., Torbeyns, J., De Smedt, B., Verschaffel, L., ... & Stern, E. (2008). A validation of eye movements as a measure of elementary school children's developing number sense. *Cognitive Development*, *23*(3), 409-422.

Segovia, I. & Castro, E. (2009). Computational and measurement estimation; curriculum foundations and research carried out at the university of granada. *Electronic Journal of Research in and Educational Psychology*, *17*(7), 499-536.

Siegel, A. W., Goldsmith, L. T., & Madson, C. R. (1982). Skill in estimation problems of extent and numerosity. *Journal for Research in Mathematics Education*, *13*(3), 211–232.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*(2), 428-444.

Siegler, R. S., & Booth, J. L. (2005). Development of numerical estimation: A review. In J. I. D. Campbell (Ed.), *The Handbook of Mathematical Cognition* (197-212). Psychology Press.

Smith, H. D. (1999). Use of the anchoring and adjustment heuristic by children. CurrentPsychology, *18*(3), 294-300.

Sowder, J., & Wheeler, M. (1989). The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education*, *20*(2), 130–146.

Sriraman, B., & Knott, L. (2009). The mathematics of estimation: Possibilities for interdisciplinary pedagogy and social consciousness. *Interchange*, *40*(2), 205-223.

Tekinkır, D. (2008). *İlköğretim 6-8.sınıf öğrencilerinin matematik alanındaki tahmin stratejilerini belirleme ve tahmin becerisi ile matematik başarısı arasındaki ilişki [To determine the estimate strategies in maths field for the primary school studens of 6th-8th grades and the relation between the estimate ability and success for maths]*. Master's thesis, Dokuz Eylül University, İzmir.

Yıldırım, A. ve Şimşek, H. (2021). *Sosyal bilimlerde nitel araştırma yöntemleri [Qualitative research methods in the social sciences]* (12th ed.). Seçkin Publishing.

Wong, T. T., Ho, C. S., & Tang, J. (2016). Consistency of response patterns in different estimation tasks. *Journal of Cognition and Development*. *17*(3), 526–547. https://doi.org/10.1080/15248372.2015.1072091