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A NOTE ON STATISTICAL APPROXIMATION PROPERTIES OF COMPLEX Q-SZÁSZ- MIRAKJAN OPERATORS

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Abstract. The complex $q-$Szász-Mirakjan operator attached to analytic functions satisfying a suitable exponential type growth condition has been studied in [14]. In this paper, we consider the A-statistical convergence of complex q-Szász- Mirakjan operator.

1. Introduction

In 1996, Phillips defined a generalization of the Bernstein operators called $q-$ Bernstein operators by using the $q-$binomial coefficients and the $q-$binomial theorem [21]. In 2008, Aral introduced $q-$Szász-Mirakjan operators and studied some approximation properties of them [12]. In 2008, Gal studied some approximation results of the complex Favard-Szász-Mirakjan operators on compact disks [17]. A different type complex $q-$Szász-Mirakjan operator was introduced by Mahmudov in [20] for $q>1$ as

$$M_{n,q}\left(f;z\right)=\sum_{k=0}^{\infty}f\left(\frac{[k]}{[n]}\right)\frac{1}{q^{k(k-1)/2}}\frac{[n]^{k}z^{k}}{[k]!}\varepsilon_{q}\left(-[n]\,q^{-k}z\right) \tag{1.1}$$

for the functions which are continuous and bounded on $[0,\infty)$.

In this paper, we study some operators by taking statistical convergence instead of ordinary convergence. In 2002, Gadjiev and Orhan gave some approximation results by using statistical convergence [1]. And several authors have studied in approximation theory by using statistical convergence concept (see [3], [4] and [5] [6], [7], [8], [13], [19]).

Now, we give some notations on $q-$analysis given in [16] and [21].
The $q-$integer $[n]$ is defined by

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$$[n]:=[n]_q=\begin{cases}\frac{1-q^n}{1-q},\ q\neq 1\\ n,\ q=1\end{cases}$$

for $q>0$ and the $q-$factorial $[n]!$ by

$$[n]!:=\begin{cases}[1]_q\,[2]_q\ldots[n]_q\,,\ n=1,2,\ldots\\ 1,\ n=0.\end{cases}$$

We give the following two $q-$analogues of the exponential function e^x which is appeared in the definition of the operator :

$$\varepsilon_q(x)=\sum_{n=0}^{\infty}\frac{1}{[n]_q!}x^n=\frac{1}{((1-q)x;\ q)_\infty},\ |x|<\frac{1}{1-q},\ |q|<1, \tag{1.2}$$

$$E_q(x)=\sum_{n=0}^{\infty}\frac{q^{\frac{n(n-1)}{2}}}{[n]_q!}x^n=\ (-(1-q)x\ ;\ q)_\infty\ ,\ x\in R,\ |q|<1, \tag{1.3}$$

where $(x;\ q)_\infty=\prod_{k=1}^{\infty}(1-xq^{k-1})$ (see [15]).

It is clear from (1.2) and (1.3) that $\varepsilon_q(x)E_q(-x)=1$ and

$$\lim_{q\to 1^-}\varepsilon_q(x)=\lim_{q\to 1^-}E_q(x)=e^x.$$

Let $q\in(0,1)\cup(1,\infty)$. The $q-$derivative of a function $f(x)$ is defined as

$$D_qf(x):=\frac{f(x)-f(qx)}{(1-q)x}\ \text{for}, x\neq 0.$$

$$D_qf(0)=\lim_{x\to 0}D_qf(x),\ \text{where}\ D_q^0f:=f,\ D_q^nf:=D_q(D_q^{n-1}f),\ n=1,2,\ldots$$

As a consequence of the definition of D_qf, we find

$$D_qx^n=[n]_q\,x^{n-1},$$
$$D_q\varepsilon_q(ax)=a\varepsilon_q(ax),$$
$$D_qE_q(ax)=aE_q(qax).$$

Also the formula for the q-differential of a product is

$$D_q(u(x)v(x))=D_q(u(x))v(x)+u(qx)D_q(v(x)).$$

We know that

$$\left(D_q(t;x)_q^n\right)(t)=[n]_q\,(t;x)_q^{n-1},$$

where $(t;x)_q^n=\prod_{k=0}^{n-1}(t-xq^k)$ (see [16]).

Now we define the complex Szász-Mirakjan operator based on $q-$integers.

Suppose that $R_{n,q} := \frac{b_n}{[n](1-q)}$, where (b_n) is a sequence of positive numbers such that $\lim_{n\to\infty} b_n = \infty$ and that $D_R = \{z \in \mathbb{C} : |z| < R\}$, $1 < R < R_{n,q}$. The complex Szász-Mirakjan operator based on $q-$integers is obtained directly from the real version (see [12]) by taking z in place of x, namely

$$\begin{aligned} S_n^q(f;z) &= S_n(f;q;z) \\ &=: E_q\left(-[n]\frac{z}{b_n}\right)\sum_{k=0}^{\infty} f\left(\frac{[k]}{[n]}b_n\right)\frac{([n]\,z)^k}{[k]!\,(b_n)^k}, \end{aligned} \tag{1.4}$$

where $n \in \mathbb{N}$, $0 < q < 1$ and $f : [R, \infty) \cup \overline{D_R} \to \mathbb{C}$ has exponential growth and it has an analytical continuation into an open disk centered at the origin.

Throughout the paper we call the operator (1.4) as complex $q-$Szász-Mirakjan operator.

It is clear that by using divided differences $S_n^q(f;z)$ can be expressed as

$$S_n^q(f;z) = S_n(f,q,z) = \sum_{j=0}^{\infty} q^{\frac{j(j-1)}{2}} f\left[0, \frac{b_n\,[1]}{[n]}, ..., \frac{b_n\,[j]}{[n]}\right] z^j, \tag{1.5}$$

similar to the real version of the $q-$Szász-Mirakjan operators (see [12]), where $f\left[0, \frac{b_n[1]}{[n]}, ..., \frac{b_n[j]}{[n]}\right]$ denotes the divided difference of f on the knots $0, \frac{b_n[1]}{[n]}, ..., \frac{b_n[j]}{[n]}$.

2. Statistical Convergence of $S_n^{q_n}(f;z)$

First of all, we recall some definitions and notations which we use in this study.

Let $A = (a_{jn})$ be a nonnegative regular matrix. The $A-$density of $K \subseteq \mathbb{N}$ given by

$$\delta_A(K) := \lim_j \sum_{n\in K}^{\infty} a_{jn},$$

whenever the limit exists. A sequence $x = (x_n)$ is called $A-$statistically convergent to a number L if for every $\varepsilon > 0$,

$$\delta_A(\{n \in \mathbb{N} : |x_n - L| \geq \varepsilon\}) = 0. \tag{2.1}$$

It is not difficult to see that (2.1) is equivalent to

$$\lim_{j\to\infty} \sum_{n:|x_n - L|\geq\varepsilon}^{\infty} a_{jn} = 0, \text{for every } \varepsilon > 0.$$

This limit expression is denoted by $st_A - \lim x_n = L$ (see in [2], [9],[10], [11]).

Now, we give a lemma which we use in the proof of Theorem 1.

Lemma 1. *Let* $D_R = \{z \in \mathbb{C} : |z| < R\}$, $1 < R < R_{n,q}$, *where* $R_{n,q} = \frac{b_n}{[n]_q(1-q)}$ *and*

$$f : [R, \infty) \cup \overline{D_R} \to \mathbb{C}$$

be continuous in $[R,\infty)\cup\overline{D_R}$*, analytic in* D_R*, namely* $f(z)=\sum_{k=0}^{\infty}c_kz^k$ *for all* $z\in D_R$ *and there exist* $M,\ C,\ B>0$ *and* $A\in\left(\frac{1}{R},1\right)$*, with the property* $|c_k|\leq\frac{MA^k}{k!}$ *for all* $k=0,1,...$*(which implies* $|f(z)|\leq Me^{A|z|}$ *for all* $z\in D_R$ *and* $|f(x)|\leq Ce^{Bx}$ *for all* $x\in[R,\infty)$*). Then* $S_n^q(f;z)$ *is well defined and analytic as function of* z *in* D_R *(see* [14]*).*

Theorem 1. *Suppose that the conditions of Lemma 1 are satisfied. Suppose also that* A *be a nonnegative regular summability matrix,* $q=q_n$ *is a sequence such that* $0<q_n<1$ *and* $st_A-\lim q_n=1$ *and* $st_A-\lim\frac{b_n}{[n]_{q_n}}=0$*.*

i. *Let* $1\leq r<\frac{1}{B}$ *be arbitrary fixed. Then for all* $|z|\leq r$*, we have*

$$st_A-\lim\left|S_n^{q_n}(f;z)-f(z)\right|=0.$$

ii. *For the simultaneous approximation by complex* $q-$*Szász-Mirakjan operator, we have*

$$st_A-\lim\left|D_{q_n}^{(p)}\left(S_n^{q_n}(f;z)\right)-D_{q_n}^{(p)}f(z)\right|=0$$

where $C_{r_1,A}$ *is given as in the case* (i)*.*

Proof. i. From [14], by taking $e_k(z)=z^k$, it is clear that $T_{n,k}(z):=S_n^{q_n}(e_k;z)$ is a polynomial of degree$\leq k$, $k=0,1,2,...$ and

$$T_{n,0}(z)=1,T_{n,1}(z)=z \text{ for all } z\in\mathbb{C}$$

Also, using $q-$derivative of $T_{n,k}(z)$ for $z\neq0$, we get

$$\begin{aligned}&D_qT_{n,k}(z)\\=&\frac{[n]_{q_n}}{zb_n}T_{n,k+1}(z)\\&-\frac{[n]_{q_n}}{b_n}E_q\left(-[n]_{q_n}\ q_n\frac{z}{b_n}\right)\sum_{j=0}^{\infty}\left(\frac{[j]_{q_n}}{[n]_{q_n}}b_n\right)^k\frac{\left([n]_{q_n}q_nz\right)^j}{[j]_{q_n}!\,(b_n)^j}\end{aligned}\tag{2.2}$$

for all $z\in\mathbb{C}$, $k=0,1,2,....$ Therefore, we obtain

$$T_{n,k}(z)=zT_{n,k-1}(q_nz)+\frac{zb_n}{[n]_{q_n}}D_q\left(T_{n,k-1}(z)\right).$$

The last equality implies that

$$\begin{aligned}&T_{n,k}(z)-z^k\\=&\frac{zb_n}{[n]_{q_n}}D_q\left(T_{n,k-1}(z)-z^{k-1}\right)+z\left[T_{n,k-1}(q_nz)-(q_nz)^{k-1}\right]\\&+\frac{[k-1]_{q_n}}{[n]_{q_n}}b_nz^{k-1}-z^k(1-q_n)\,[k-1]_{q_n}.\end{aligned}\tag{2.3}$$

From the Bernstein inequality in $\overline{D_r} = \{z \in \mathbb{C}:\ |z| \leq r\}$, we have

$$|D_q(P_k(z)| \leq \|P_k'\| \leq \frac{k}{r}\|P_k\|_r\,, \tag{2.4}$$

where $\|.\|_r = \max_{z\in\overline{D_r}} |f(z)|$ (see [18, p. 55]). From (2.3) and (2.4), we obtain that

$$\begin{aligned} &\left|T_{n,k}(z) - z^k\right| \\ \leq\ & \frac{rb_n}{[n]_{q_n}} \left\|T_{n,k-1}(z) - z^{k-1}\right\|_r \frac{k-1}{r} \\ &+ r\left|T_{n,k-1}(q_n z) - (q_n z)^{k-1}\right| + r^{k-1}\frac{[k-1]_{q_n}}{[n]_{q_n}} b_n + r^k\,[k-1]_{q_n}\,|1-q_n|\,. \end{aligned}$$

By passing to norm we reach to

$$\begin{aligned} &\left\|T_{n,k}(z) - z^k\right\|_r \\ \leq\ & \left(\frac{(k-1)b_n}{[n]_{q_n}} + r\right) \left\|T_{n,k-1}(z) - z^{k-1}\right\|_r + r^k k\left(1 - q_n + \frac{b_n}{[n]_{q_n}}\right). \end{aligned}$$

By using mathematical induction with respect to k, the above recurrence formula gives that

$$\left\|T_{n,k}(z) - z^k\right\|_r \leq \frac{(k+1)!r^k}{2}\left(1 - q_n + \frac{b_n}{[n]_{q_n}}\right)$$

for all $k \geq 2$ and fixed an arbitrary $n \geq n_0$. There exists an n_0 such that for all $n > n_0$, then $\frac{b_n}{[n]_{q_n}} < 1$. Assume that it is true for k. Since $[k]_{q_n} \leq (k+1)$ is satisfied for all $0 < q_n < 1$, the recurrence formula reduces to

$$\begin{aligned} &\left\|T_{n,k+1}(z) - z^{k+1}\right\|_r \\ \leq\ & \left(1 - q_n + \frac{b_n}{[n]_{q_n}}\right)\frac{r^{k+1}}{2}\left\{(k+1)!k\frac{b_n}{[n]_{q_n}} + (k+1)! + 2(k+1)\right\} \end{aligned}$$

for all $k \geq 2$ and for all $n > n_0$. By this inequality, it follows

$$\left\|T_{n,k+1}(z) - z^{k+1}\right\|_r \leq \frac{(k+2)!}{2} r^{k+1}\left(1 - q_n + \frac{b_n}{[n]_{q_n}}\right).$$

for $k \geq 2$ and for all $n > n_0$.

Now, we show that

$$S_n^{q_n}(f;z) = \sum_{k=0}^{\infty} c_k S_n^{q_n}(e_k;z) = \sum_{k=0}^{\infty} c_k T_{n,k}(z) \tag{2.5}$$

for all $z \in D_R$. For any $m \in \mathbb{N}$, let us define

$$f_m(z) = \sum_{j=0}^{m} c_j z^j \text{ if } |z| \leq r < R \text{ and } f_m(x) = f(x) \text{ if } x \in (r, \infty).$$

From the hypothesis on f, it is clear that for any $m \in \mathbb{N}$, $|f_m(x)| \leq C_m e^{B_m x}$ for all $x \in [0,\infty)$. Ratio test implies that for each fixed m, $n \in \mathbb{N}$ and z,

$$|S_n^{q_n}(f_m;z)| \leq C_m \left| E_q\left(-[n]_{q_n}\frac{z}{b_n}\right)\right| \sum_{k=0}^{\infty}\frac{\left([n]_{q_n}\right)^k |z|^k}{[k]_{q_n}!\,(b_n)^k} e^{B_m\left(\frac{[k]_{q_n}}{[n]_{q_n}}b_n\right)} < \infty.$$

Therefore, $S_n^{q_n}(f_m;z)$ is well defined. Now, we set

$$f_{m,k}(z) = c_k e_k(z) \text{ if } |z| \leq r \text{ and } f_{m,k}(x) = \frac{f(x)}{m+1} \text{ if } x \in (r,\infty).$$

It is clear that each $f_{m,k}$ is of exponential growth on $[0,\infty)$ and that

$$f_m(z) = \sum_{k=0}^{m} f_{m,k}(z).$$

Since $S_n^{q_n}$ is linear, we have

$$S_n^{q_n}(f_m;z) = \sum_{k=0}^{m} c_k S_n^{q_n}(e_k;z) \text{ for all } |z| \leq r,$$

which proves that

$$\lim_{m\to\infty} S_n^{q_n}(f_m;z) = S_n^{q_n}(f;z)$$

for any fixed $n \in \mathbb{N}$ and $|z| \leq r$. But this is immediate from

$$\lim_{m\to\infty} \|f_m - f\|_r = 0$$

and from the inequality

$$\begin{aligned} |S_n^{q_n}(f_m) - S_n^{q_n}(f)| &\leq \left| E_q\left(-[n]_{q_n}\frac{z}{b_n}\right)\right| \varepsilon_q\left([n]_{q_n}\frac{|z|}{b_n}\right)\|f_m - f\|_r \\ &\leq M_{r,n}\|f_m - f\|_r, \end{aligned}$$

for all $|z| \leq r$. Consequently the statement (2.5) is satisfied.

In this way, from the hypothesis on c_k, this implies for all $|z| \leq r$

$$\begin{aligned} &|S_n^{q_n}(f;z) - f(z)| \\ \leq\ & \left(1 - q_n + \frac{b_n}{[n]_{q_n}}\right) C_{r,B}, \end{aligned} \tag{2.6}$$

where

$$C_{r,B} = \frac{MA}{2}\sum_{k=2}^{\infty}(k+1)(rA)^{k-1}$$

is finite for all $1 \leq r < \frac{1}{B}$. Note that the series $\sum_{k=2}^{\infty} u^{k+1}$ and its derivative $\sum_{k=2}^{\infty}(k+1)u^k$ are uniformly and absolutely convergent in any compact disk included in the open unit disk.

As $st_A - \lim q_n = 1$ there exists $n_1(\varepsilon)$ and $K_1 \subseteq \mathbb{N}$ of density 1 such that $1 - q_n < \varepsilon$ for all $n \in K_1$ and $n > n_1(\varepsilon)$. On the other hand, since $st_A - \lim \frac{b_n}{[n]_{q_n}} = 0$ there exists $n_2(\varepsilon)$ and $K_2 \subseteq \mathbb{N}$ of density 1 such that $\frac{b_n}{[n]_{q_n}} < \varepsilon$ for all $n \in K_2$ and $n > n_2(\varepsilon)$. Now define $K = K_1 \cap K_2$. Note that $\delta_A(K_1 \cap K_2) = 1$ and for all $\varepsilon > 0$ and for all $n > n_0(\varepsilon) = \max\{n_1, n_2\}$

$$1 - q_n + \frac{b_n}{[n]_{q_n}} < \varepsilon \tag{2.7}$$

Hence (2.7) and (2.6) imply that

$$st_A - \lim |S_n^{q_n}(f; z) - f(z)| = 0.$$

ii. Let γ be the circle of radius $r_1 > r$ with centered 0, since for any $|z| \leq r$ and $v \in \gamma$, we have $|v - z| \geq r_1 - r$, by Cauchy's formulas it follows that for all $|z| \leq r$ and $n \in \mathbb{N}$

$$\begin{aligned}
\left| D_{q_n}^{(p)} \left(S_n^{q_n}(f; z) \right) - D_{q_n}^{(p)} f(z) \right| &\leq \left| S_n^{q_n^{(p)}}(f; z) - f^{(p)}(z) \right| \\
&= \frac{p!}{2\pi} \left| \int_\gamma \frac{S_n^{q_n}(f; v) - f(v)}{(v - z)^{p+1}} dv \right| \\
&\leq \left(1 - q_n + \frac{b_n}{[n]_{q_n}} \right) C_{r_1, B} \frac{p!}{2\pi} \frac{2\pi r_1}{(r_1 - r)^{p+1}} \\
&= \left(1 - q_n + \frac{b_n}{[n]_{q_n}} \right) C_{r_1, B} \frac{p! r_1}{(r_1 - r)^{p+1}}.
\end{aligned}$$

Similarly we get from hypothesis that for all $\varepsilon > 0$ there exists a subset $K \subseteq \mathbb{N}$ of density 1 and $n_0 = n_0(\varepsilon)$ such that $\left| D_{q_n}^{(p)} \left(S_n^{q_n}(f; z) \right) - D_{q_n}^{(p)} f(z) \right| < \varepsilon$ for all $n > n_0$ and $n \in K$. □

Remark 1. *Consider the matrix method $C = (c_{jn})$ which is called Cesåro matrix and defined as*

$$c_{jn} = \begin{cases} \frac{1}{j}, & n \leq j \\ 0, & \text{otherwise} \end{cases}.$$

In this case A-statistical convergence reduces to statistical convergence. Now define a sequence $q = (q_n)$ as

$$q_n = \begin{cases} \frac{1}{n}, & n = m^2 \ (m \in \mathbb{N}) \\ \frac{n}{n+1}, & \text{otherwise} \end{cases}.$$

It is obvious that q is not convergent but it is statistically convergent to 1.

Remark 2. *Let d_n be a sequence of positive numbers such that $d_n \to \infty$ and $\frac{d_n}{[n]_{q_n}} \to 0$ as $n \to \infty$. Note that the sequence defined as*

$$b_n = \begin{cases} n, & n = m^2 \ (m \in \mathbb{N}) \\ d_n, & \text{otherwise} \end{cases},$$

$\left(\frac{b_n}{[n]_{q_n}}\right)$ *is statistically convergent to zero.*

Note that these examples do not satisfy the hypothesis of Theorem 2.3 in [14], but they satisfy the hypothesis of our theorem.

Remark 3. *If we take $A = I$ identity matrix, we get ordinary convergence. Therefore when we take $A = I$, we have Theorem 2.3 in* [14].

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