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# A Weighted Algorithm for Solving a Cauchy Problem of The Sideways Parabolic Equation

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Abstract. In this paper, a weighted algorithm based on the reduced differential transform method is presented for solving some sideways parabolic equations. The proposed approach uses initial and boundary conditions simultaneously for obtaining an approximate analytical solution of equation. A description of the algorithm to solve the problem and determining the boundary condition is given. Finally, some examples are discussed to show ability of the presented algorithm and to confirm utility of this method.

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**Keywords:** Weighted reduced differential transform method, sideways parabolic equation, inverse problem.

## 1. Introduction

Convection-diffusion equations occur in various fields of applications. Mathematical models of many natural phenomena are presented by using this type of equations for example processes in fluid mechanics, astrophysics, meteorology, multiphase flow in oil reservoirs, polymer flow, financial modeling, and several other areas [1, 9, 11, 13]. Thus, computing solutions of these equations is very important. Researchers in literature have used different methods for solving this type of equations [1, 4, 9, 11, 13, 16].

In many industrial applications, the temperature on one side of a thick wall is determined, but the other side is inaccessible to measurements [5, 8]. In a one-dimensional case, this problem leads to the following parabolic equation [12]

$$u_t = a(x)u_{xx} + b(x)u_x + g(x,t) \qquad 0 < x < L,\ 0 < t < T, \tag{1.1}$$

with the conditions

$$u(x,0) = f(x), \qquad 0 \le x \le L, \tag{1.2}$$

$$u(0,t) = \phi(t), \qquad 0 \le t \le T, \tag{1.3}$$

$$u_x(0,t) = \psi(t), \qquad 0 \le t \le T, \tag{1.4}$$

where $a(x)$, $b(x)$, $f(x)$, $\phi(t)$, $\psi(t)$ and $g(x,t)$ are given functions. The boundary function $\varphi(t)$, the solution at $x = L$, is unknown.

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For example the above problem is a simple model of the heat transfer inside a thermocouple of a suction pyrometer. The suction pyrometer [7], is used for controlling and measuring the gas temperature in a combustion chamber. In this case, equation (1.1) is called the sideways heat equation. The problem (1.1)- (1.4) is a type of inverse problems belongs to the class of parabolic equations. These type of problems have been investigated by researchers in many scientific works [6, 7, 10, 12, 17, 19, 20].

In this work, we present the weighted reduced differential transform method (WRDTM) to solve the Cauchy problem (1.1)-(1.4). The reduced differential transform method (RDTM) has been used by many authors to obtain analytical and approximate solutions to nonlinear problems [2, 3, 14, 15, 21]. Here we introduce an algorithm based on the RDTM to solve the inverse problems of (1.1)-(1.4) types. The differential transformation method which has been applied in most of literature, is started by the initial condition. But for our purposes, we use the initial and boundary conditions. The problem is solved twice. at first by initial condition, then by boundary conditions. A weighted combination of two solutions is considered for final solution of the problem.

The arrangement of this paper is in the following plan: Section 2 is a review of the RDTM. In Section 3, a weighted algorithm is introduced. Finally, in Section 4, some test problems are solved in order to show the ability and efficiency of the algorithm.

## 2. Reduced Differential Transform Method

In present section, the definitions and operations of RDTM will be reviewed. Suppose that a two variables function $u(x,t)$ is separable as $u(x,t) = p(x)q(t)$. We can represent this function as

$$u(x,t) = \sum_{i=0}^{\infty} P_i x^i \sum_{j=0}^{\infty} Q_j t^j = \sum_{r=0}^{\infty} U_r(x) t^r = \sum_{r=0}^{\infty} V_r(t) x^r,$$

according to the features of differential transform [21] where $U_r(x)$ and $V_r(t)$ are called t-dimensional and x-dimensional spectrum functions of $u(x,t)$, respectively.

**Definition 2.1.** Suppose $u(x,t)$ is analytic and differentiated continuosly with respect to $x$ and $t$ in their domains. Then

i) The transformed function $U_r(x)$ is defined as

$$U_r(x) = \frac{1}{r!}\left[\frac{\partial^r}{\partial t^r} u(x,t)\right]_{t=0}. \tag{2.1}$$

Also, its inverse differential transformation of will be as

$$u(x,t) = \sum_{r=0}^{\infty} U_r(x) t^r. \tag{2.2}$$

ii) The transformed function $V_r(t)$ is defined as

$$V_r(t) = \frac{1}{r!}\left[\frac{\partial^r}{\partial x^r} u(x,t)\right]_{x=0}.$$

The inverse differential transformation of $V_r(t)$ will be as

$$u(x,t) = \sum_{r=0}^{\infty} V_r(t) x^r.$$

The main operations of the reduced differential transformation, according to the variable $t$, are listed in Table 1, that can be deduced from Eqs. (2.1) and (2.2) [2, 14]. These operations can be obtained similarly for reduced differential transforms according to the variable $x$.

## 3. The WRDTM Solution

Now, a weighted method according to the RDTM (WRDTM) will be presenteded to solve (1.2)-(1.4). We perform the algorithm in two steps. At the first step, consider (1.1) and suppose $L = \frac{\partial}{\partial t}$. By applying basic properties of differential transformations and Table 1, the differential transformation of (1.1) with the condition (1.2) becomes

$$(r+1)U_{r+1}(x) = a(x)\frac{\partial^2}{\partial x^2} U_r(x) + b(x)\frac{\partial}{\partial x} U_r(x) + G_r(x), \tag{3.1}$$

TABLE 1. Some operations of the reduced differential transform.

| Function Form | Transformed Form |
|---|---|
| $u(x,t)$ | $U_r(x) = \frac{1}{r!}\left[\frac{\partial^r}{\partial t^r}u(x,t)\right]_{t=0}$ |
| $u(x,t) = d$ ($d$ is a constant) | $U_r(x) = \delta(r) = \begin{cases} 1 & r=0 \\ 0 & r\neq 0 \end{cases}$ |
| $u(x,t) = v(x,t) + w(x,t)$ | $U_r(x) = V_r(x) + W_r(x)$ |
| $u(x,t) = cv(x,t)$ | $U_r(x) = cV_r(x)$ ($c$ is a constant) |
| $u(x,t) = x^m v(x,t)$ | $U_r(x) = x^m V_r$ |
| $u(x,t) = t^m v(x,t)$ | $U_r(x) = V_{r-m}$ |
| $u(x,t) = x^m t^n$ | $U_r(x) = x^m\delta(r-n) = \begin{cases} x^m & r=n \\ 0 & r\neq n \end{cases}$ |
| $u(x,t) = \frac{\partial^m}{\partial t^m}v(x,t)$ | $U_r(x) = \frac{(r+m)!}{r!}V_{r+m}(x)$ |
| $u(x,t) = \frac{\partial^m}{\partial x^m}v(x,t)$ | $U_r(x) = \frac{\partial^m}{\partial x^m}V_r(x)$ |

where $G_r(x)$ is the transformation of $g(x,t)$. By substituting of the $U_0(x) = f(x)$ as differential transformation of (1.2) into (3.1), the approximate solution

$$\hat{u}_n(x,t) = \sum_{r=0}^{n} U_r(x)t^r. \tag{3.2}$$

will be obtained.

At the second step, we seek the series solution of the Eq. (1.1) according to conditions (1.3) and (1.4). Suppose $L = \frac{\partial^2}{\partial x^2}$. Taking the differential transformation of (1.1) and applying basic properties of differential transformation in Table 1 with respect to x, we get

$$\frac{\partial}{\partial t}V_r(t) = (r+1)(r+2)V_l(t) + (r+1)V_m(t) + G_r(t), \tag{3.3}$$

where $l$ and $m$ are natural numbers related to $r$. From the boundary conditions (1.3) and (1.4), we have

$$V_0(t) = \phi(t), \tag{3.4}$$

and

$$V_1(t) = \psi(t). \tag{3.5}$$

Substituting (3.4) and (3.5) into (3.3), we can get the successive values of $V_r(t)$. In result, the series solution

$$\check{u}_n(x,t) = \sum_{r=0}^{n} V_r(t)x^r \tag{3.6}$$

will be obtained.

Now, the approximate solution for the sideways heat equation (1.1) with conditions (1.2), (1.3) and (1.4) will be considered as a weighted combination

$$u_n(x,t) = c\hat{u}_n(x,t) + (1-c)\check{u}_n(x,t), \tag{3.7}$$

where $c$ is a constant on the interval $[0,1]$. For determining the best value of $c$ for each $n$, we use the idea presented in [18].

**Theorem 3.1.** *Suppose that $f(x) \in L^2[(0,L)]$, $\phi(t), \psi(t) \in L^2[(0,T)]$ and $\|.\|$ denotes the $L^2-$norm. Let*

$$c_1 = \|\hat{u}_n(0,t) - \phi(t)\|,$$
$$c_2 = \|\frac{\partial \hat{u}_n}{\partial x}(0,t) - \psi(t)\|,$$
$$c_3 = \|\check{u}_n(x,0) - f(x)\|.$$

*Then the best value for c in (3.7) is*

$$c = \frac{c_3^2}{c_1^2 + c_2^2 + c_3^2}, \qquad n \geq 0.$$

*Proof.* According to conditions (1.2)-(1.4), we define the following residual function for $0 \le x \le L$ and $0 \le t \le T$ as

$$F_n(x,t;c) = \|u_n(0,t) - \phi(t)\| + \|\frac{\partial u_n}{\partial x}(1,t) - \psi(t)\| + \|u_n(x,0) - f(x)\|. \tag{3.8}$$

Substituting (3.7) into (3.8), we have

$$\begin{aligned} F_n(x,t;c) \quad = \quad & \|c\hat{u}_n(0,t) + (1-c)\check{u}_n(0,t) - \phi(t)\|^2 \\ & + \|c\frac{\partial \hat{u}_n}{\partial x}(1,t) + (1-c)\frac{\partial \check{u}_n}{\partial x}(1,t) - \psi(t)\|^2 \\ & + \|c\hat{u}_n(x,0) + (1-c)\check{u}_n(x,0) - f(x)\|^2. \end{aligned}$$

From (3.2), (3.6) and (3.7), we get

$$\begin{aligned} F_n(x,t;c) \quad = \quad & \|c\hat{u}_n(0,t) + (1-c)\phi(t) - \phi(t)\|^2 + \|c\frac{\partial \hat{u}_n}{\partial x}(1,t) + (1-c)\psi(t) - \psi(t)\|^2 \\ & + \|cf(x) + (1-c)\check{u}_n(x,0) - f(x)\|^2 \\ = \quad & \|c\hat{u}_n(0,t) - c\phi(t)\|^2 + \|c\frac{\partial \hat{u}_n}{\partial x}(1,t) - c\psi(t)\|^2 \\ & + \|(1-c)\check{u}_n(x,0) - (1-c)f(x)\|^2 = c^2c_1^2 + c^2c_2^2 + (1-c)^2c_3^2. \end{aligned}$$

The best value of $c$ will minimize the residual function $F_n$. Thus, by setting the partial derivative of $F_n$ with respect to $c$ equal to zero, we find

$$c = \frac{c_3^2}{c_1^2 + c_2^2 + c_3^2}, \qquad n \ge 0.$$

□

For evaluating the unknown boundary condition $\varphi(t)$ on $x = 1$, the approximate solution (3.7) will be used.

## 4. Illustrative Examples

To show the applicability of the WRDTM, some examples will be presented. We use $n$ terms in evaluating the approximate solution $u_n(x,t)$.

**Example 4.1.** Let us consider $a(x) = 1$, $b(x) = -x$, $g(x,t) = -e^{-x}(1+x)$, $f(x) = e^{-x} + x$, $\phi(t) = 1$ and $\psi(t) = -1 + e^{-t}$ on $Q \equiv \{(x,t)|(x,t) \in [0,1] \times [0,1]\}$. With these assumptions, the problem (1)–(4) has the solution $u(x,t) = e^{-x} + xe^{-t}$ and $\varphi(t) = e^{-t} + \frac{1}{e}$. Using the properties of the differential transformation with respect to $t$, we can write

$$(r+1)U_{r+1}(x) = \frac{\partial^2}{\partial x^2}U_r(x) - x\frac{\partial}{\partial x}U_r(x) - e^{-x}(1+x),$$

or

$$U_r(x) = \frac{1}{r}\frac{\partial^2}{\partial x^2}U_{r-1}(x) - x\frac{\partial}{\partial x}U_{r-1}(x) - e^{-x}(1+x).$$

By substituding of $U_0(x) = e^{-x} + x$ as transformation of the initial condition according to mathematical induction, we find

$$U_r(x) = \frac{(-1)^r x}{r!}, \qquad r \ge 1.$$

The inverse differential transform of $U_r(x)$ gives:

$$\begin{aligned} \hat{u}_n(x,t) = & \sum_{r=0}^{n} U_r(x)t^r \\ = & e^{-x} + x - xt + \frac{xt^2}{2} - \frac{xt^3}{6} + \frac{xt^4}{24} + \cdots + \frac{(-1)^n xt^n}{n!}. \end{aligned}$$

In other sides, for $m \geq 5$, $c = 1$, therefore, from (3.7) the solution is obtained as

$$u(x,t) = \lim_{n\to+\infty} \hat{u}_n(x,t) = \sum_{r=0}^{\infty} \frac{(-1)^r x}{r!} t^r = e^{-x} + xe^{-t}.$$

Finally, by using this solution, the unknown boundary condition at $x = 1$ will be get as $u(1,t) = \varphi(t) = e^{-t} + \frac{1}{e}$.

**Example 4.2.** Consider the problem

$$\begin{aligned}
&\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x}, && 0 \leq x \leq 1, 0 \leq t \leq 1 \\
&u(x,0) = e^x - x, && 0 \leq x \leq 1, \\
&u(0,t) = 1 + t, && 0 \leq t \leq 1, \\
&u_x(0,t) = 0, && 0 \leq t \leq 1.
\end{aligned} \tag{4.1}$$

The exact solution of this problem is [18]:

$$u(x,t) = e^x - x + t.$$

To apply WRDTM, taking the differential transform of (4.1), gives

$$U_{r+1}(x) = \frac{1}{r+1}\Big(\frac{\partial^2}{\partial x^2} U_r(x) - \frac{\partial}{\partial x} U_r(x)\Big). \tag{4.2}$$

By substituding of $U_0(x) = e^x - x$ into (4.2), it is find that,

$$U_1(x) = 1,$$

and

$$U_r(x) = 0, \qquad r \geq 2.$$

Therefore, we obtain

$$\hat{u}_n(x,t) = \sum_{r=0}^{n} U_r(x) t^r = e^x - x + t.$$

Now, we take differential transformation of (4.1) with respect to $x$, i.e.

$$U_{r+2}(t) = \frac{1}{(r+1)(r+2)}\Big(\frac{\partial}{\partial t} U_r(t) + (r+1) U_{r+1}(t)\Big). \tag{4.3}$$

Substituding of $U_0(t) = 1 + t$ and $U_1(t) = 0$ into (4.3) gives

$$U_r(t) = \frac{1}{r!}, \qquad r \geq 2.$$

Then

$$\check{u}_n(x,t) = 1 + t + \tfrac{x^2}{2} + \tfrac{x^3}{6} + \tfrac{x^4}{24} + \cdots + \tfrac{x^r}{r!}$$

For $m \geq 10$ we get $c = 0$, Thus we have

$$u(x,t) = \lim_{n\to+\infty} \check{u}_n(x,t) = e^x - x + t.$$

In result, by using this solution, we get $\varphi(t) = t + e - 1$.

**Example 4.3.** Let us consider the problem (1)–(4) with $a(x) = 1$, $b(x) = 2$, $g(x,t) = x$, $f(x) = e^{-x}\cos(x) + 0.125e^{-2x} + 0.25x(1-x) - 1$, $\phi(t) = e^{-2t} - 0.875$ and $\psi(t) = -e^{-2t}$ on $Q = \{(x,t)|(x,t) \in [0,1] \times [0,5]\}$. With these assumptions, this problem has the exact solution $u(x,t) = e^{-2t-x}\cos(x) + 0.125e^{-2x} + (\frac{x}{4})(1-x) - 1$ and $\varphi(t) = \cos(1)e^{-2t-1} + 0.125e^{-2} - 1$.

With respect to $t$, the differential transformation of Eq. (1.1) becomes

$$U_{r+1}(x) = \frac{1}{r+1}\Big(\frac{\partial^2}{\partial x^2} U_r(x) + 2\frac{\partial}{\partial x} U_r(x) + \delta(r).x\Big), \tag{4.4}$$

where $\delta(r)$ is the Dirac delta function.

Substituting of $U_0(x) = e^{-x}\cos(x) + 0.125e^{-2x} + 0.25x(1-x) - 1$ into (4.4) gives

$$
\begin{aligned}
U_1(x) &= -2e^{-x}cos(x),\\
U_2(x) &= 2e^{-x}cos(x),\\
U_3(x) &= -\frac{4}{3}e^{-x}cos(x),\\
U_4(x) &= \frac{2}{3}e^{-x}cos(x),\\
&\vdots
\end{aligned}
$$

Now, we take the differential transformation of the Eq. (1.1) with respect to $x$. Again we apply the properties of differential transformation in Table 1 and obtain

$$\frac{\partial}{\partial t}U_r(t) = (r+1)(r+2)U_{r+2}(t) + 2(r+1)U_{r+1}(t) + \delta(r-1),$$

or

$$U_r(t) = \frac{1}{r(r-1)}\left(\frac{\partial}{\partial t}U_{r-2}(t) - 2(r-1)U_{r-1}(t) - \delta(r-3)\right). \tag{4.5}$$

After substituting $U_0(t) = e^{-2t} - 0.875$ and $U_1(t) = -e^{-2t}$ as transformation of boundary conditons (1.3) and (1.4), into (4.5), we obtain the next terms as

$$
\begin{aligned}
U_2(t) &= 0,\\
U_3(t) &= \frac{1}{6}(-1+2e^{-2t}),\\
U_4(t) &= \frac{1}{12}(1-2e^{-2t}),\\
U_5(t) &= \frac{1}{30}(-1+2e^{-2t}),\\
U_6(t) &= \frac{1}{90}, U_7(t) = \frac{1}{630}(-2-e^{-2t}),\\
&\vdots
\end{aligned}
$$

Therefore, the series solution can be obtained by (3.2).
The relative error of the approximate solutions at $t = 2.5$ for different values of $n$ are given in Table 2.

TABLE 2. The relative error for Example 4.3 when $t = 2.5$.

| $x$ | $n=10$ | $n=15$ | $n=20$ | $n=25$ | $n=30$ |
|---|---|---|---|---|---|
| 0.1 | $5.2963E-3$ | $4.8109E-5$ | $6.1769E-8$ | $2.0888E-9$ | $3.3379E-10$ |
| 0.2 | $4.7110E-3$ | $4.2793E-5$ | $5.4943E-8$ | $1.8580E-9$ | $2.9323E-11$ |
| 0.3 | $4.1394E-3$ | $3.7599E-5$ | $4.8276E-8$ | $1.6325E-9$ | $2.5846E-11$ |
| 0.4 | $3.5883E-3$ | $3.2594E-5$ | $4.1849E-8$ | $1.4151E-9$ | $2.2515E-11$ |
| 0.5 | $3.0648E-3$ | $2.7839E-5$ | $3.5744E-8$ | $1.2087E-9$ | $1.9105E-11$ |
| 0.6 | $2.5751E-3$ | $2.3391E-5$ | $3.0032E-8$ | $1.0156E-9$ | $1.6258E-11$ |
| 0.7 | $2.1244E-3$ | $1.9296E-5$ | $2.4774E-8$ | $8.3775E-10$ | $1.3166E-12$ |
| 0.8 | $1.7166E-3$ | $1.5588E-5$ | $2.0014E-8$ | $6.7677E-10$ | $1.0880E-12$ |
| 0.9 | $1.3546E-3$ | $1.2288E-5$ | $1.5778E-8$ | $5.3351E-10$ | $8.4853E-12$ |

Now, the approximate solution and its accuracy is given for a value of n with more detail. For example, Suppose

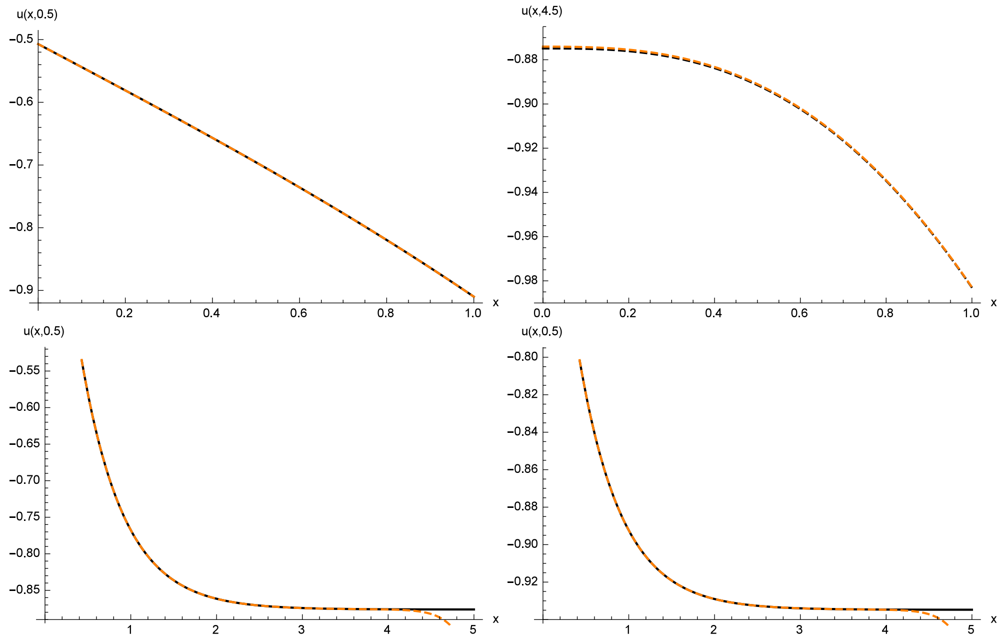


Figure 1. The exact solution (Black) and the approximate solutions (Orange) of Example 4.3 with $n = 25$ for various values of $t$ and $x$.

$n = 25$. So, we will have

$$\begin{aligned}\hat{u}_{25}(x,t) &= \sum_{r=0}^{25} U_r(x)t^r \\ &= -1 + 0.125e^{-2x} + 0.25x(1-x) \\ &\quad + e^{-x}cos(x)(1 - 2t + 2t^2 - 1.33333t^3 + \cdots \\ &\quad + 2.70405 \times 10^{-17}t^{24} - 2.16324 \times 10^{-18}t^{25}),\end{aligned}$$

and

$$\begin{aligned}\check{u}_{25}(x,t) &= \sum_{r=0}^{25} U_r(x)x^r \\ &= -0.875 + e^{-2t} - e^{-2t}x + \frac{-1+2e^{-2t}}{6}x^3 \\ &\quad + \frac{1-2e^{-2t}}{12}x^4 + \cdots + \frac{512+e^{-2t}}{6.6\times 10^{21}}x^{24} - \frac{1024+e^{-2t}}{2.64\times 10^{22}}x^{25}.\end{aligned}$$

According to Theorem 3.1, we get $c = 0.2054$. Now, the approximate solution will be obtained by (3.7). The exact solution of problem and its approximations are shown in Figure 1, for various values of $t$ and $x$. Exact boundary function at $x = 1$ and its approximation are shown in Figure 2. Finally Figure 3 exhibits the error function of the approximate solution for domain $Q$.

## 5. Conclusion

In this work, a Cauchy problem of a sideways parabolic equation was considered. In this problem one of boundary conditions was unknown. By using the RDTM, a weighted algorithm was introduced to determine the solution of the sideways equation and the unkonwn boundray condition. To show the capability and reliability of the method, some numerical test examples were presented in the last section. The results verify that the WRDTM is an efficient technique to solve such problems.

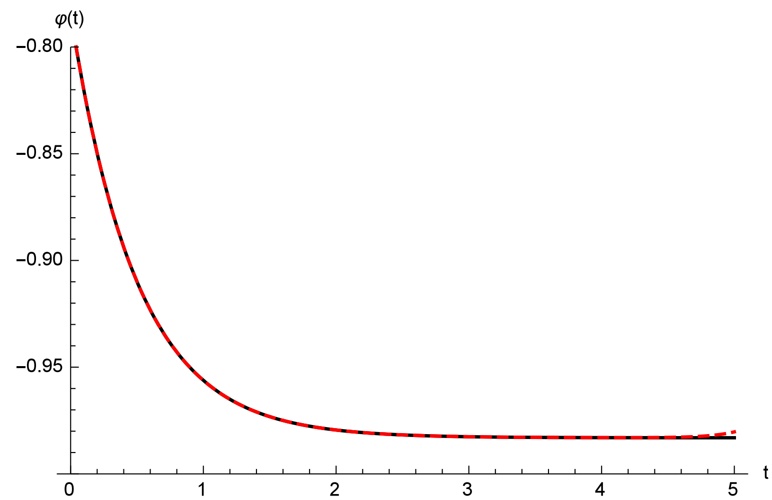


FIGURE 2. The exact boundary condition (Black) and its approximation (Red) for Example 4.3.

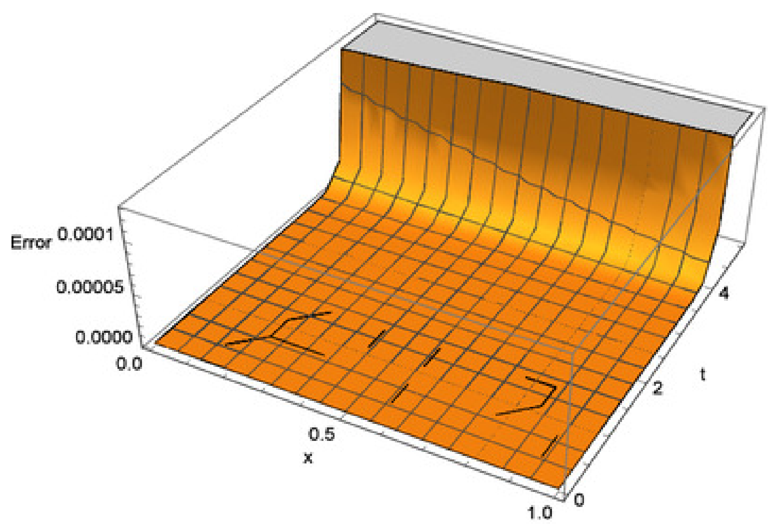


FIGURE 3. The error function of the approximate solution for Example 4.3.

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