

## A Study of Modified Nanofluid Flow Over an Exponentially Stretching Surface With Inclined Magnetic Field and Porous Media

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### ABSTRACT

A numerical study of  $Fe_3O_4 - TiO_2 - Ni/C_2H_6O_2$  modified nanofluid's flow through a stretched surface is presented in the current work with an applied angled magnetic field. In the subsequent form of hybrid nanofluid, known as modified nanofluid, three distinct suspended nanoparticles in a base fluid are taken into consideration. Iron Oxide, Nical, and Titanium Dioxide nanoparticles are suspended in ethanol glycol, which is used as a base liquid. One way to improve heat transfer rates in MHD flow over a stretched surface with variable viscosity is to utilize modified nanofluids. This is useful in a number of sectors, including energy systems, thermal management in aircraft, and cooling electronic systems. By applying the proper similarity transformations, the Runga-Kutta fourth order technique encounters the mathematical framework of the flow. One important finding is that, in contrast to nanofluids and hybrid nanofluids, the modified nanofluid has a larger capacity for heat transmission. The modified nanofluid's heat transfer capabilities exhibit intriguing behavior that calls for more research on it. There are numerical solutions that are displayed graphically. Increases in the volume fraction parameter and the inclination angle parameter of the magnetic field have been found to cause a drop in the velocity field of the modified nanofluid.

**Keywords:** Heat Transfer; MHD; Modified Nanofluid; Porous Media; Stretching Sheet

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### 1. Introduction

Non-Newtonian fluids have been applied in several industrial processes in recent years. Numerous sectors, including the pharmaceutical, chemical, biological, and petroleum industries, are benefiting from the unique qualities of non-Newtonian fluids. Non-Newtonian fluids have a wide range of uses, which has led to a fast growth in their study. In many industrial applications, regular fluids like water, ethylene-glycol, propylene-glycol, and engine-oil are used for heat transmission. Enhancing these liquids' ability to transport heat can lower the cost of goods, power, processing time, and size, and increase device operating time. The fact that traditional heat exchangers require low temperatures to operate is one issue with heat exchange systems. The thermal conductivity of this liquid may be optimized with the distribution of solid particles. Thermal conductivity of heterogeneous two-components system was investigated by R.

Hamilton [1]. The thermal conductivities of a variety of heterogeneous two-component systems were studied by researchers in 1962. The empirical shape factor, which is dependent on the thermal conductivities of the phases and the included particle's shape, can be determined with accuracy using an equation. A review on nanofluid was written by W. Yu and H. Xie [2]. Preparations and stability factors were discussed by the authors in their review. They also discuss the opportunity for future of nanofluid. Nanofluids square measure dispersion of nano-materials (such as nano-fibers, nano-tubes, nanoparticles nano-rods, nano-wires, nano-sheets, or droplets) in fundamental fluids. Due in large part to the early observations of aberrant thermal physical phenomena (k) advancement of nanofluids with any low fraction of nanoparticles, nanofluids have garnered considerable interest over the past ten years. With numerous reports of unusual improvements in thermal physical phenomena and numerous



Table 1. Thermo-physical characteristics of modified nanofluid: [8,10]

$\mu_{mnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}(1 - \phi_3)^{2.5}}$	Effective dynamic viscosity
$\rho_{mnf} = (1 - \phi_3) \left\{ [(1 - \phi_1)\rho_f + \phi_1\rho_{s1}](1 - \phi_2) \right\} + \phi_2\rho_{s2} + \phi_3\rho_{s3},$	Effective density
$\sigma_{mnf} = \frac{2\sigma_{hnf} + \sigma_{s3} - 2(\sigma_{hnf} - \sigma_{s3})\phi_3}{2\sigma_{hnf} + \sigma_{s3} + (\sigma_{hnf} - \sigma_{s3})\phi_3} \sigma_{hnf}$ where	Electrical conductivity
$\sigma_{hnf} = \frac{2\sigma_{nf} + \sigma_{s2} - 2(\sigma_{nf} - \sigma_{s2})\phi_2}{2\sigma_{nf} + \sigma_{s2} + (\sigma_{nf} - \sigma_{s2})\phi_2} \sigma_{nf}$	
and $\sigma_{nf} = \frac{\sigma_{s1} + 2\sigma_f - 2\phi_1(\sigma_f - \sigma_{s1})}{\sigma_{s1} + 2\sigma_f + \phi_1(\sigma_f - \sigma_{s1})} \sigma_f$	
$\kappa_{mnf} = \frac{\left\{ \frac{\kappa_{s3} + (n-1)\kappa_{hnf} - (n-1)(\kappa_{hnf} - \kappa_{s3})\phi_3}{\kappa_{s3} + (n-1)\kappa_{hnf} + (\kappa_{hnf} - \kappa_{s3})\phi_3} \right\} \kappa_{hnf}}{\kappa_{s2} + (n-1)\kappa_{nf} - (n-1)(\kappa_{nf} - \kappa_{s2})\phi_2}$ where	Thermal conductivity
$\kappa_{hnf} = \frac{\left\{ \frac{\kappa_{s2} + (n-1)\kappa_{nf} - (n-1)(\kappa_{nf} - \kappa_{s2})\phi_2}{\kappa_{s2} + (n-1)\kappa_{nf} + (\kappa_{nf} - \kappa_{s2})\phi_2} \right\} \kappa_{nf}}{\kappa_{s1} + (n-1)\kappa_f - (n-1)(\kappa_f - \kappa_{s1})\phi_1}$	
$(\rho C_p)_{mnf} = (1 - \phi_3) \left\{ (1 - \phi_2) \left[ (1 - \phi_1)(\rho C_p)_f + (\rho C_p)_{s1} \phi_1 \right] + (\rho C_p)_{s2} \phi_2 \right\} + \phi_3(\rho C_p)_{s3},$	Heat capacitance

Table 2 Thermo-physical values: [2, 6, 7, 11]

	$\rho$ (kg /m <sup>3</sup> )	$C_p$ (J /kg K)	$k$ (W /m K)	$\sigma$ (S/m)
$C_2H_6O_2$	1116.6	2382	0.249	0.01485
$Fe_3O_4$	5200	670	9.8	$0.74 \times 10^6$
$Ni$	8900	444	90.7	$1.7 \times 10^7$
$TiO_2$	4175	692	8.4	$6.27 \times 10^{-5}$

**2.1. Similarity transformation**

To solve our model, we apply the similarity transformation below[11]:

$$u = U_0 e^{\frac{x}{l}} f'(\eta), v = -\sqrt{\frac{\nu_f U_0}{2l}} e^{x/2l} [f(\eta) + \eta f'(\eta)]$$

and  $(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$  (5)

Where  $\eta = y \sqrt{\frac{U_0}{2lv_f}} e^{x/2l}$  and using the aforementioned transformations, the nonlinear equations of the flow have been incorporated into the subsequent ordinary differential equations.

$$A f''' + B(f f'' - 2f'^2) - \frac{\sigma_{mnf}}{\sigma_f} M \sin^2 \alpha f' = 0. \tag{6}$$

$$\frac{1}{Pr} \frac{\kappa_{mnf}}{\kappa_f} C \theta'' + f \theta' - f' \theta = 0. \tag{7}$$

Here

$$A = \frac{1}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}(1 - \phi_3)^{2.5}},$$

$$B = (1 - \phi_3) \left\{ (1 - \phi_2) \left[ (1 - \phi_1) + \phi_1 \frac{\rho_{s1}}{\rho_f} \right] + \phi_2 \frac{\rho_{s2}}{\rho_f} \right\} + \phi_3$$

and

$$C = \left[ (1 - \phi_3) \left\{ (1 - \phi_2) \left[ (1 - \phi_1) + \phi_1 \frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right] + \phi_2 \frac{(\rho C_p)_{s2}}{(\rho C_p)_f} \right\} + \phi_3 \frac{(\rho C_p)_{s3}}{(\rho C_p)_f} \right]^{-1}$$

Boundary condition (5) is also modified using the aforementioned transformations as:

$$\theta(0) = 1, f(0) = 0, f'(0) = 1, \theta(\infty) = 0, f'(\infty) = 0. \tag{8}$$

Where  $M = \frac{2lB_0^2 \sigma_f}{U_0 \rho_f e^{x/l}}$  is Magnetic parameter,  $Pr = \frac{(\rho C_p)_f \nu_f}{\kappa_f}$  is

Prandtl number. An additional physical measure is the local Nusselt number, which is provided by

$$Nu_x = -\left( \frac{\kappa_{hnf}}{\kappa_f} + \frac{4R}{3} \right) \psi_1 x \theta'(0). \tag{9}$$

**3. Numerical Solution**

Moreover, Eq. (6) and Eq. (7) and boundary conditions Eq. (8) have been recast as first-order initial value problems, using the following definitions:

$$f' = h_2, f'' = h_3, \theta = h_4, \theta' = h_5, h'_3 = \frac{B}{A} (2h_2^2 - h_1 h_3) +$$

$$\frac{M \sigma_{mnf}}{A \sigma_f} h_2 \sin^2 \alpha \text{ and } h'_5 = \frac{Pr \cdot \kappa_f}{C \kappa_{mnf}} (h_2 h_4 - h_1 h_5).$$

With boundary condition,  $h_1(0) = 0, h_2(0) = 1, h_3(0) = 1, h_4(0) = 1.$

For numerical solutions these ODE's are converted into first ordered IVP. In this IVP only three initial conditions are present, but five are required for the solution as  $h_3(0)$  and  $h_5(0)$ . Assuming the initial guess values for  $h_3(0)$  and  $h_5(0)$  and suitable finite value of  $\eta(\rightarrow \infty)$ , say  $\eta_\infty$  numerical solutions by Runge-Kutta method are obtained. Then computations for  $f'(\eta)$  and  $\theta(\eta)$  at  $\eta_\infty (\cong 10)$  along with conditions of boundary  $f'(\eta_\infty) = 0$  and  $\theta(\eta_\infty) = 0$  are performed. Approximated (degree of accuracy is  $10^{-6}$ ) solutions have obtained by adjusting the values of  $f''(0)$  and  $\theta'(0)$  by considering step size  $\Delta \eta = 0.01$ . This technique uses an iterated strategy to provide accurate findings up to  $10^{-7}$  accuracy.

### 4. Result and Discussions

The flow characterization through an exponentially stretched sheet of modified nanofluids is investigated. Various inclination angles are used while applying a magnetic field. Further, the variation of several contributing parameters on the fluid velocity and temperature profiles are presented via graphs.

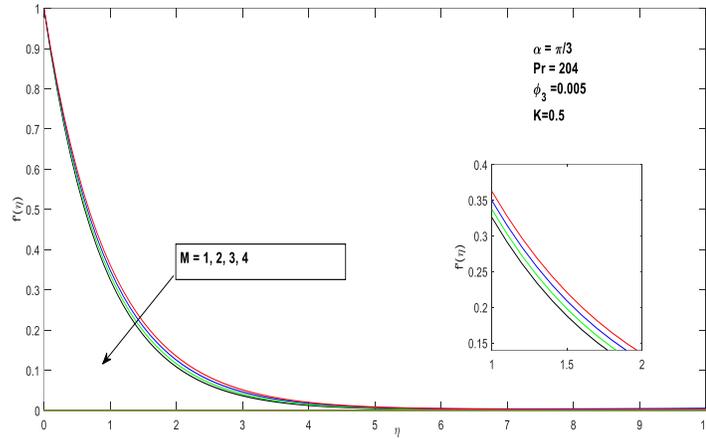


Figure 2. Velocity profile for the magnetic field parameter M.

The consequence of M on  $f'(\eta)$  for the modified nanofluid  $Fe_3O_4 - Ni - TiO_2/C_2H_6O_2$  is shown in Figure 2. The velocity profile decreases as M increases, according to the findings. The magnetic field produced by the movement of an electrically conducting fluid creates a Lorentz force, which has the retardation feature. This is the reason for the diminishing appearance of the velocity profile. As M rises, the retardation strength of all the three nanofluids grows. As a result, velocity decreases as well.

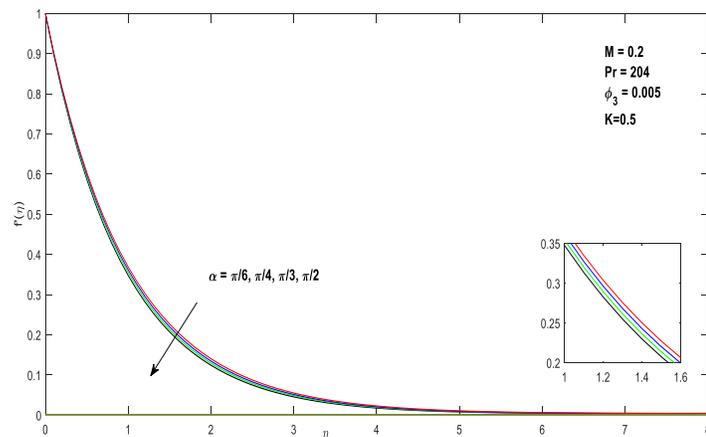


Figure 3. Velocity profile for the inclination angle parameter  $\alpha$ .

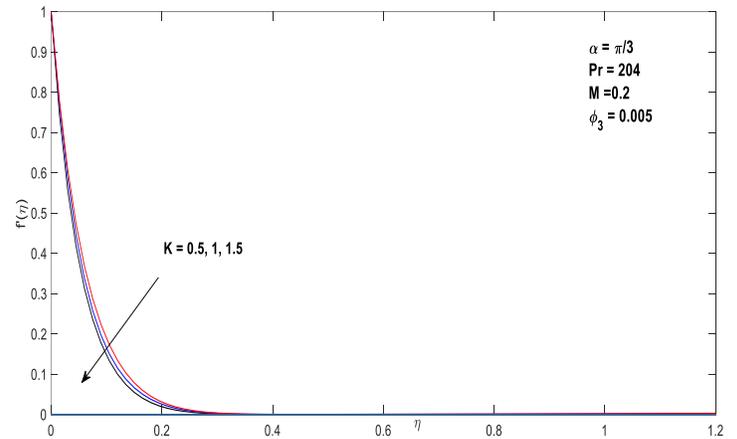


Figure 4. Velocity profile for the permeability parameter K.

Figure 3 shows the impact of the  $\alpha$  on  $f'(\eta)$ . When the parameter is increased, the  $f'(\eta)$  profile falls. This is due to the fact that as the angle of inclination increases, the magnetic field becomes stronger and the Lorentz force reduces the velocity field. The influence of the permeability parameter K on the velocity profile is illustrated in Figure 4. A distinct decline in velocity within the flow pattern is noticed. The impact of volume friction  $\phi_3$  on  $f'(\eta)$  is shown in Figure 5.

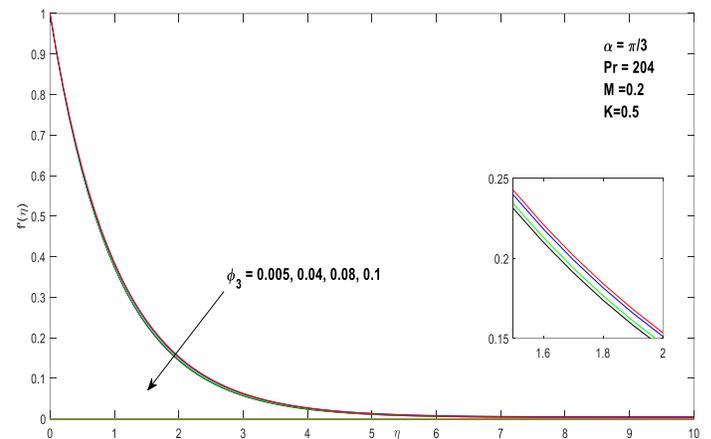


Figure 5. Velocity profile for the parameter  $\phi_3$ .

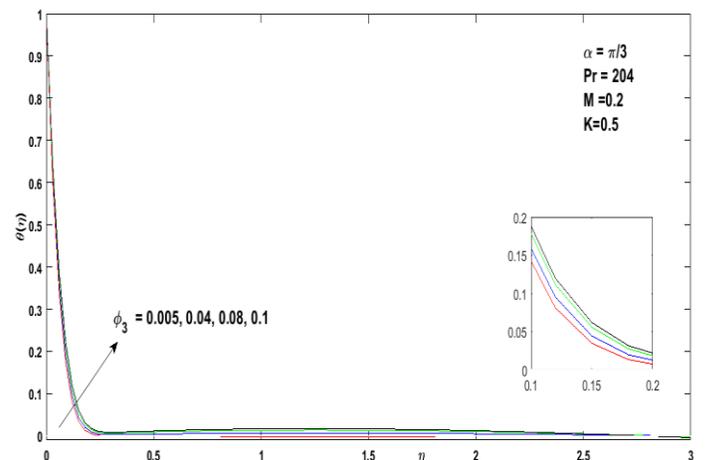


Figure 6. Temperature profile for the parameter  $\phi_3$ .

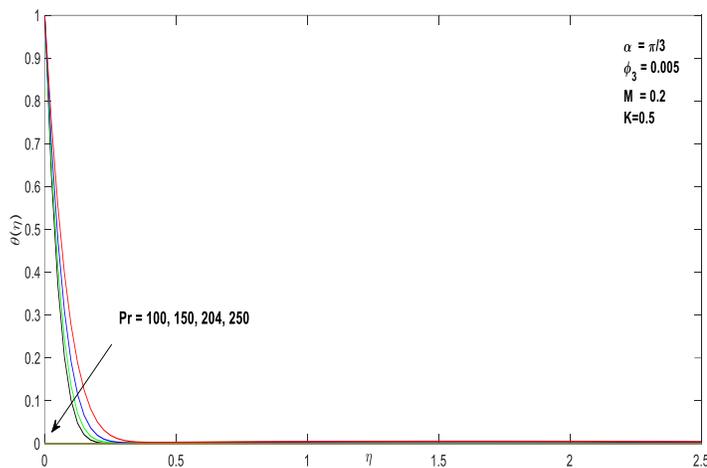


Figure 7. Temperature profile for the parameter  $\phi_3$ .

It is seen that a drop in the velocity profile occurs with an increase in  $\phi_3$ . Additionally, it is noted for  $\phi_3$  that the fluid's speed increases with increasing  $\phi_3$  away from the sheet and decreases in its immediate vicinity. Figure 6 illustrates how  $\phi_3$  affects  $\theta(\eta)$ .

It has been shown that for changed nanofluids, an enhanced temperature profile results from increased  $\phi_3$ . A higher temperature field may arise from the extraction of more energy from a greater number of nanoparticles. The influence of Pr on  $\theta(\eta)$  is shown in Figure 7. The figures indicate that a reduction in the temperature profile  $\theta(\eta)$  is noticed with an increase in Pr.

## 5. Conclusions

The impact of heat transfer of modified nanofluids flowing past an exponentially extending surface under the influence of an applied angled magnetic field has been studied. A modified nanofluid composed of ethylene glycol that contains a suspension of  $Fe_3O_4 - TiO_2 - Ni$  nanoparticles have been visually displayed. By applying the proper similarity transformations, the Runge-Kutta fourth order technique solves the flow's governing models.

The key results that follow are achieved.

- A decrease in the velocity field has been observed for the modified nanofluid with an increase in the inclination angle and magnetic field parameter.
- The velocity curve for  $\phi_3$  shows that the fluid velocity increases away from the sheet and reduces toward the wall.
- A decrease is observed with an increase in the velocity field's  $\phi_3$ , but a reversal of this impact is shown in the temperature profile.

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## Conflict of Interest Statement

The authors declare that there is no conflict of interest in the study.

## CRediT Author Statement

**N. Jain:** Conceptualization, Mathematical modeling, Writing-original draft **M. Gaur:** Conceptualization, Supervision **Priyanka Agrawal:** Validation, Software, result analysis **P.K. Dadheech:** Data curation, Formal analysis, revisions.

## References

1. Hamilton, R. L., & Crosser, O. K. (1962). Thermal conductivity of heterogeneous two-component systems. *Industrial & Engineering chemistry fundamentals*, 1(3), 187-191.
2. Yu, W., & Xie, H. (2012). A review on nanofluids: preparation, stability mechanisms, and applications. *Journal of nanomaterials*, 2012(1), 435873.
3. Hemmat Esfe, M., Saedodin, S., Wongwises, S., & Toghraie, D. (2015). An experimental study on the effect of diameter on thermal conductivity and dynamic viscosity of Fe/water nanofluids. *Journal of Thermal Analysis and Calorimetry*, 119, 1817-1824.
4. Imtiaz, M., Hayat, T., Alsaedi, A., & Ahmad, B. (2016). Convective flow of carbon nanotubes between rotating stretchable disks with thermal radiation effects. *International journal of heat and mass transfer*, 101, 948-957.
5. Chamkha, A. J., Dogonchi, A. S., & Ganji, D. D. (2019). Magneto-hydrodynamic flow and heat transfer of a hybrid nanofluid in a rotating system among two surfaces in the presence of thermal radiation and Joule heating. *Aip Advances*, 9(2).
6. Acharya, N., Maity, S., & Kundu, P. K. (2020). Influence of inclined magnetic field on the flow of condensed nanomaterial over a slippery surface: the hybrid visualization. *Applied Nanoscience*, 10(2), 633-647.
7. Waini, I., Ishak, A., & Pop, I. (2019). Flow and heat transfer along a permeable stretching/shrinking curved surface in a hybrid nanofluid. *Physica Scripta*, 94(10), 105219.
8. Dadheech, P. K., Agrawal, P., Mebarek-Oudina, F., Abu-Hamdeh, N. H., & Sharma, A. (2020). Comparative heat transfer analysis of MoS<sub>2</sub>/C<sub>2</sub>H<sub>6</sub>O<sub>2</sub> and SiO<sub>2</sub>-MoS<sub>2</sub>/C<sub>2</sub>H<sub>6</sub>O<sub>2</sub> nanofluids with natural convection and inclined magnetic field. *Journal of Nanofluids*, 9(3), 161-167.
9. Nadeem, S., & Abbas, N. (2019). Effects of MHD on modified nanofluid model with variable viscosity in a porous medium. *Nanofluid Flow in Porous Media*, 7.
10. Abbas, N., Nadeem, S., & Issakhov, A. (2021). Transportation of modified nanofluid flow with time dependent viscosity over a Riga plate: exponentially stretching. *Ain Shams Engineering Journal*, 12(4), 3967-3973.
11. Dadheech, P. K., Agrawal, P., Sharma, A., Nisar, K. S., & Purohit, S. D. (2021). Transportation of Al<sub>2</sub>O<sub>3</sub>-SiO<sub>2</sub>-TiO<sub>2</sub> modified nanofluid over an exponentially stretching surface with inclined magnetohydrodynamic. *Thermal Science*, 25(Spec. issue 2), 279-285.