

Equivalent Stress Analysis of Functionally Graded Rectangular Plates by Genetic Programming

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Abstract

In this study, the sets of equation were extracted by Genetic Programming (GP) for thermal stress analysis of one-dimensional functionally graded rectangular plates. First, thermal stress analyses were performed using a finite difference method for a sufficient number of compositional gradient exponents. Then, equation sets were obtained by the GP using the maximum and minimum equivalent stress levels obtained from these analyses. Appropriate models are produced for equivalent stress levels at compositional gradient exponents. The models achieved these levels 100 times faster than the finite difference method by using GP. GP provided significant time gain in deriving sets of equations for thermal stress analysis of plates with current boundary conditions.

Keywords: “Functionally graded plates, genetic programming, finite difference methods, thermal stress analysis.”

1. Introduction

Materials in materials science and technology are being designed and investigated for new material properties arising from the necessity. For this purpose, functionally graded materials are considered to be both high thermal resistance and thermal stress resistant materials for high temperature applications [1-4]. These materials are designed on the one side as a metal, on the other side as a function of volume and transition region volume fraction. Functionally change of the transition region prevents the occurrence of stress discontinuities which can be caused by the incompatibilities of the thermal and mechanical properties of the two different materials, namely ceramic and metal transition regions. In these special materials, it is important to determine the material composition, that is, the compositional gradient exponents, for which the thermal discontinuities are the lowest for different thermal and structural boundary conditions [5-7]. Thus, cracks or breaks in these regions can be prevented.

The main applications of different algorithms [8-15] in the literature having same purpose are separated to provide solutions for different boundary conditions and different mechanical effects and to determine optimum compositional gradient upper value. Goupee and Vel [16] performed the optimization of the natural frequencies of bidirectional functional graded beams. They used the Genetic Algorithms (GA) to optimize the material composition. Ashjari and Khoshrovan [17] performed mass optimization for strain distributions in the bending test of functionally graded materials. They used GA and particle swarm optimization in their work. They compared the convergence speed of these algorithms and the convergence accuracy. Karaboğa et al. [18] proposed a model with the artificial bee colony algorithm. They compared the results of the method with GP. In [19] it is investigated the applicability of layer optimization for the Artificial Bee Colony algorithm to maximize the lowest fundamental frequency of symmetric laminate composite plates. They compared the results with the GA. Apalak et al. presented layer optimization of plates using the GA to maximize the lowest fundamental frequency of plates of layered composites [20]. Tahami et al. [21] investigated the optimum design by evaluating the sudden temperature changes at the nodes of the functional cascaded material. They presented their experiments with the GA to provide the optimum volume distribution. Nguyen and Lee [22] investigated the optimum design of functionally graded beams under buckling. They used the GA to determine the optimum design according to the thickness of the beams.

Guse and Brezocnik [23] presented analyzes of tensile strength and conductivity by cold-formed materials with the GP. They emphasized that the results of the GP are quite good and the cost of design was reduced. Brezocnik [24] developed different prediction models by the GP for the cold tensile strength of alloyed bars in different conditions. Ali has proved its validity by comparing the test model with the best model that is determined by the GP. Vassilopoulos et al. [25] used the GP to measure the

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durability of the reinforced fiber composite material. They indicated that the modeling technique they offer with the GP is more suitable than traditional methods.

Ahishek et al. [26] used the GP to optimize the processing performance of polymer composite materials during manufacturing. Patterns is compared with GP and Adaptive Neuro Fuzzy. Russo et al. [27] used the GP to increase productivity during manufacture. They have determined optimal working conditions by the GP for the problems they deal with. Gandomi and Alavi [28] are proposed GP method for modeling structural engineering problems and determined optimum conditions for various complex structure problems. They presented their models by comparing them with the literature. In the literature, studies have been made to optimize or estimate the compositional gradient exponents, which determines the material composition of many functionally graded materials. However, in the studies related to FGM and GP, there is no study in which the grading is along the plane and the finite difference method used as the numerical solution method. In this respect, our work will provide important contributions to literature.

In this study, the sets of equation were presented by the GP for thermal stress analysis of one-dimensional functionally graded rectangular plates. First, thermal stress analyses were performed [29] program using a finite difference method for a sufficient number of compositional gradient exponents. Then, equation sets were obtained by the GP using the maximum and minimum equivalent stress levels obtained from this analysis. As a result, produced appropriate models to obtain equivalent stress levels at compositional gradient exponents. The models achieved these levels are 100 times faster than the finite difference method. GP provided significant time gain in deriving sets of equations for thermal stress analysis of plates with current boundary conditions

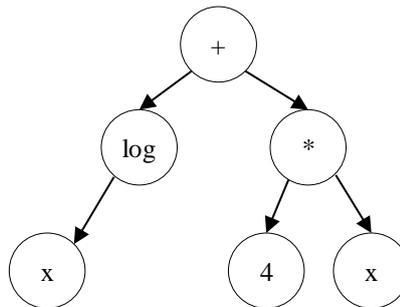
2. Material and Method

2.1. Functionally Graded Materials

FGM are designed on the one side as a metal, on the other side as a function of volume and transition region volume fraction. These materials are intended for high temperature applications. Because of the discontinuity of the material in the transition regions between layers in conventional composites, interfacial cracks occur. By using graded materials, the discontinuities in the interfacial area were removed and interfacial defects and cracks were prevented. In these materials, it is very important to determine the optimum compositional gradient exponents in terms of workability of the material within economic, functionality and elastic boundaries.

2.2. Yapı

Genetic Programming, developed by Koza [30], has been applied to solve numerous interesting problems [31-35]. Genetic Programming which uses the same analogy as GA is a most well-known automatic programming technique. The basic steps for the GP method are similar to the steps of GA. The most important difference GP and GA is in the representation of individuals. While GA express individuals as fixed code sequences, GP express them parse trees. The trees are randomly generated according to tree depth which is previously determined. The production of tree nodes is provided by terminals (constants or variables such as x , y , 5) and functions (arithmetic operators such as $+$, $-$, $*$, $/$). Nodes create branches and branches form solution trees. The mathematical relationship of the solution model in GP can be represented by individuals which is described in Equation 1 as given in Figure 1. In these notations, x is used to represent the independent, and $f(x)$ is dependent.



$$f(x) = \log(x) + 4x \quad (1)$$

Figure 1: Representation of an example solution in GP with tree structure

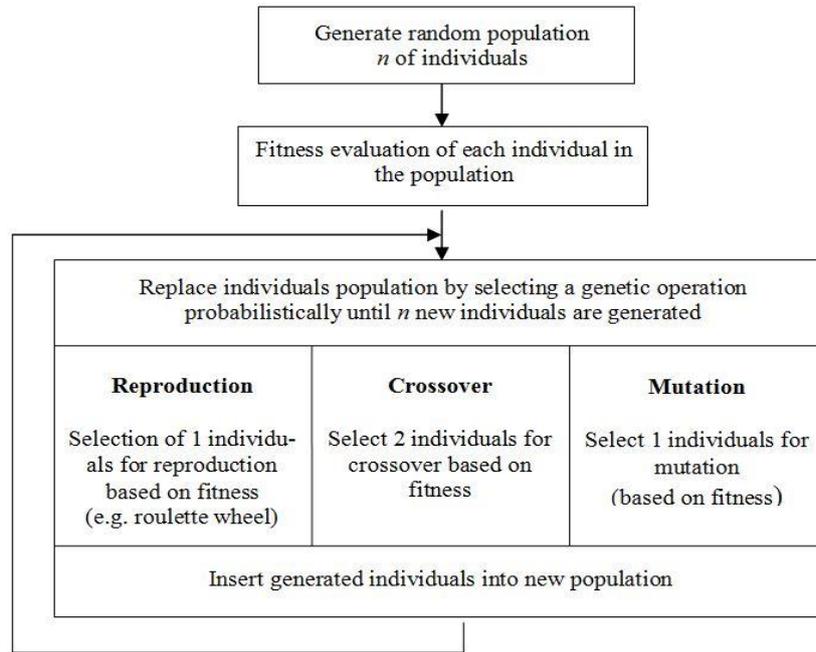


Figure 2: The flow chart of Genetic Programming.

A flow chart of genetic programming is given in Figure 2 [36]. The first step in the flow chart is the generating the initial population. Individuals are produced by the full method, the grow method, or the ramped half and half method [37]. The quality of each solution tree is determined by taking into account the predetermined fitness function of each problem. Individuals with high quality are more likely to pass on to the next generation. Genetic programming iteratively transforms a population of computer programs into a new generation of the population by applying genetic operations such as reproduction, crossover and mutation. These operations are applied to individual(s) selected from the population. Choosing the best individuals according to fitness is applied with methods like tournament, roulette wheel [38]. The crossover operator allows the hybrid of two selected individuals to produce a new individual. Generally, sub-trees taken from two crossing points selected from parent trees are crossed to obtain new hybrid trees. The mutation operator provides unprecedented and unexplored individual elements [39]. With elitism, it is ensured that the good generations of the previous generation are transferred to the current generation. The method is terminated when it is met stopping criteria such as the specific fitness value of the individuals or the number of generations.

2.3. Problem Description and Experimental Design

In this section, the sets of equation were presented using Genetic Programming (GP) for thermal stress analysis of one-dimensional functionally graded rectangular plates. First, thermal stress analyses were performed using a finite difference method for a sufficient number of compositional gradient exponents. Numerical methods were used to obtain the true values for graded upper values of 200 different composition randomly generated in the range [0.001-1.5]. Then, equation sets were obtained by the GP using the maximum and minimum equivalent stress levels obtained from this analysis. In the problem, while the plate was subjected to an in-plane constant heat flux $q = 50 \text{ kW/m}^2$ at $y = h$ along its ceramic edge, the heat flux at the other edges was zero and adiabatic boundary conditions were assumed. The initial temperature was taken as 298 K for the whole plate and analysis was completed when the temperature reached 600 K at any point on the metal edge opposite the edge of the heat flux. Since the 1 mm plate thickness was much smaller than the other dimensions, the strain and stress in the thickness direction were neglected and 2-D analyses were performed. The plate was fixed along all its edges. The plate edge exposed to heat flux is completely ceramics (ZrO_2). The edge opposite this edge is completely metal (Ti-6Al-4V).

Datasets

This section demonstrates feature selected classification ability of GP, set of experiments conducted. In this study, each dataset is split into a training set and test set to investigate performance of extracted models in GP. The number of features, training instances and test instances of the four datasets are shown in Table 1. All datasets are almost split with 70% of instances randomly selected from the datasets for training and the other 30% instances forms test set.

Table 1. Characteristics of the datasets considered in the experiments

Dataset	Total Instances	Training Instances	Test Instances
$(\sigma_{eqv})_1$ (the greatest of the greatest value of equivalent stress levels)	200	140	60
$(\sigma_{eqv})_2$ (the smallest value of the largest value of the equivalent stress levels)	200	140	60
$(\sigma_{eqv})_3$ (the largest value of the smallest value of the equivalent stress levels)	200	140	60
$(\sigma_{eqv})_4$ (the smallest value of the smallest value of the equivalent stress levels)	200	140	60

Fitness Function- Parameters

The performance of models obtained by GP is evaluated by the Root Mean Square Error (RMSE) on both the training set and the test set. The fitness function is shown Eq. (2).

$$fitness = \sqrt{\frac{\sum_1^n (y_{pred} - y_{actual})^2}{n}} \quad (2)$$

Where n define the data size, y_{actual} is y values from data set, y_{pred} is the estimated y value obtained by entering the values of the solution set of the obtained solution. The complexity of the obtained solution is calculated as in Eq. (3) in proportion to the depth of the tree and the number of nodes.

$$C = \sum_{k=1}^d n * k \quad (3)$$

Where C is tree complexity, d is the depth of the solution tree and n is the number of nodes at depth. The parameters for GP is summarized in Table 2. for the problems. The same parameters are used for all data sets.

Table 2. Parameters

Parameters	GP
Population Size	100
Generation	100
Maximum Tree Depth	4
Crossover Rate	0.14
Mutation Rate	0.84
Direct Reproduction Rate	0.02
Initialization	Ramped Half and Half
Functions	+, -, *

3. Results and Discussion

This section demonstrates symbolic regression abilities of GP, set of experiments conducted.

3.1. Simulation Results

The experiments are run 100 times independently and the obtained training results are demonstrated in Table 3 and test results are in Table 4. Standard deviation in the tables is shown as S.D. As seen in Table 3, the most error value was $(\sigma_{eqv})_2$ with

4.66 value. Best fitness was achieved using $(\sigma_{\text{eqv}})_1$. The maximum standard deviation is found $(\sigma_{\text{eqv}})_3$ with 7.53 value in dataset. Extracted model which produces the closest value to the real values is found in $(\sigma_{\text{eqv}})_2$ with 4.66 fitness.

Table 3. Training Simulation Results

Dataset	Criteria	Metric
$(\sigma_{\text{eqv}})_1$	Mean	9.39
	Max	10.04
	Min	8.52
	S.D	0.38
$(\sigma_{\text{eqv}})_2$	Mean	11.14
	Max	15.22
	Min	4.66
	S.D	2.22
$(\sigma_{\text{eqv}})_3$	Mean	14.08
	Max	14.4
	Min	7.53
	S.D	1.02
$(\sigma_{\text{eqv}})_4$	Mean	23.40
	Max	24.11
	Min	13.66
	S.D	1.93

Figure 3 ($(\sigma_{\text{eqv}})_1$), Figure 4 ($(\sigma_{\text{eqv}})_2$), Figure 5 ($(\sigma_{\text{eqv}})_3$), Figure 6 ($(\sigma_{\text{eqv}})_4$) are shown training model predictions and actual values and Figure 7 ($(\sigma_{\text{eqv}})_1$), Figure 8 ($(\sigma_{\text{eqv}})_2$), Figure 9 ($(\sigma_{\text{eqv}})_3$), Figure 10 ($(\sigma_{\text{eqv}})_4$) are shown test model predictions and actual values Good predictions are those close to the identity line. As seen in the graphs, the predicted values are close to the actual values.

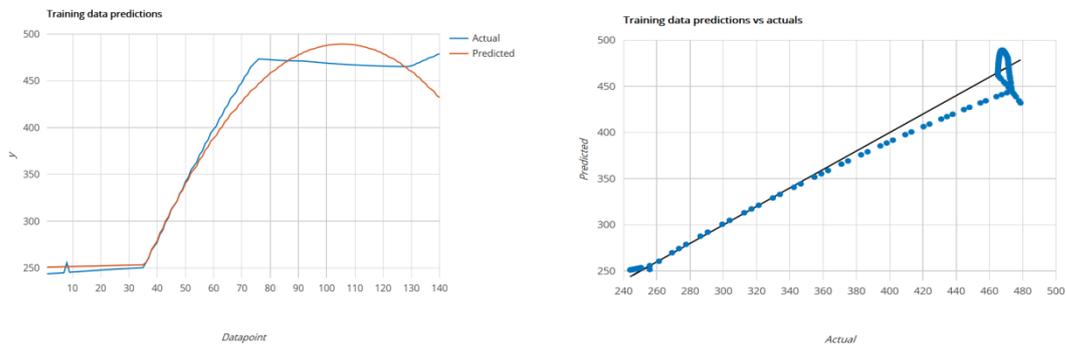


Fig 3. Training Actual with Predicted Model Graph($\sigma_{\text{eqv}})_1$

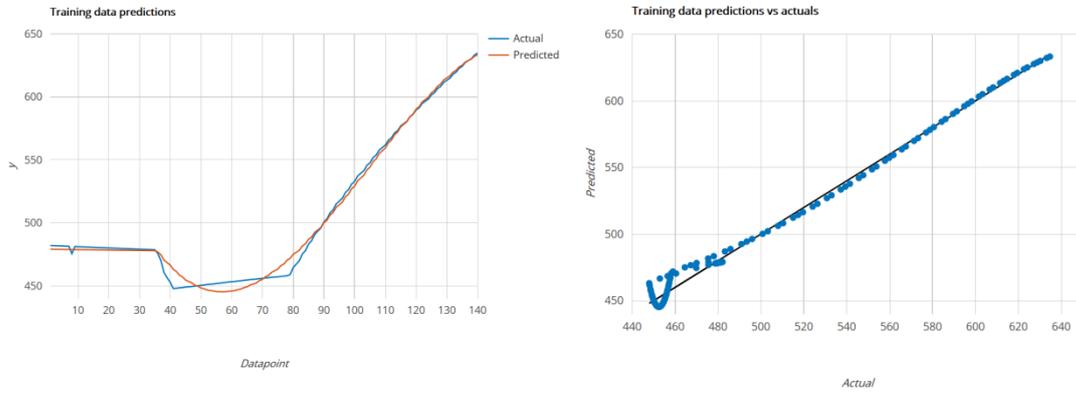


Fig 4. Training Actual with Predicted Model Graph(σ_{eqv})₂



Fig 5. Training Actual with Predicted Model Graph(σ_{eqv})₃

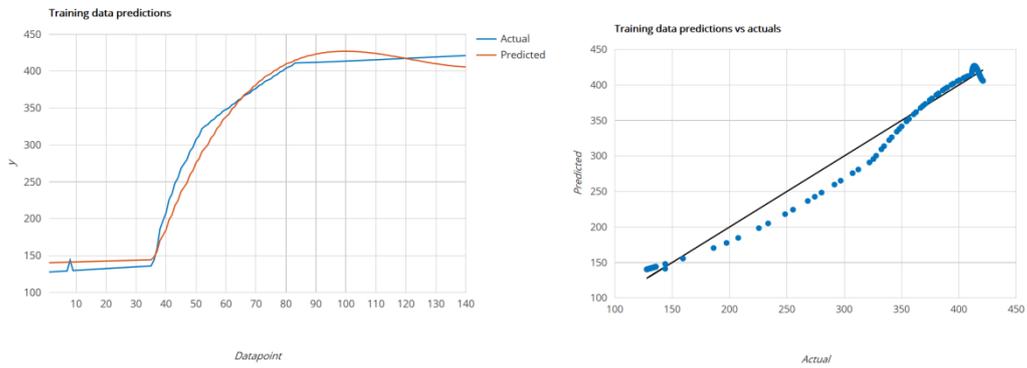


Fig 6. Training Actual with Predicted Model Graph(σ_{eqv})₄

Table 4. Test Simulation Results

Dataset	Criteria	Metric
$(\sigma_{\text{eqv}})_1$	Mean	9.45
	Max	10.08
	Min	8.54
	S.D	0.39
$(\sigma_{\text{eqv}})_2$	Mean	11.42
	Max	15.47
	Min	4.67
	S.D	2.24
$(\sigma_{\text{eqv}})_3$	Mean	14.86
	Max	15.22
	Min	7.77
	S.D	1.09
$(\sigma_{\text{eqv}})_4$	Mean	23.86
	Max	24.6
	Min	13.69
	S.D	2.00

As shown in Table 4, the average value in the test data set is 9.45 in $(\sigma_{\text{eqv}})_1$ and the worst fitness value has 24.6 in $(\sigma_{\text{eqv}})_4$. The model which is minimum error 4.67 fitness are produced equivalent stress values in $(\sigma_{\text{eqv}})_2$.

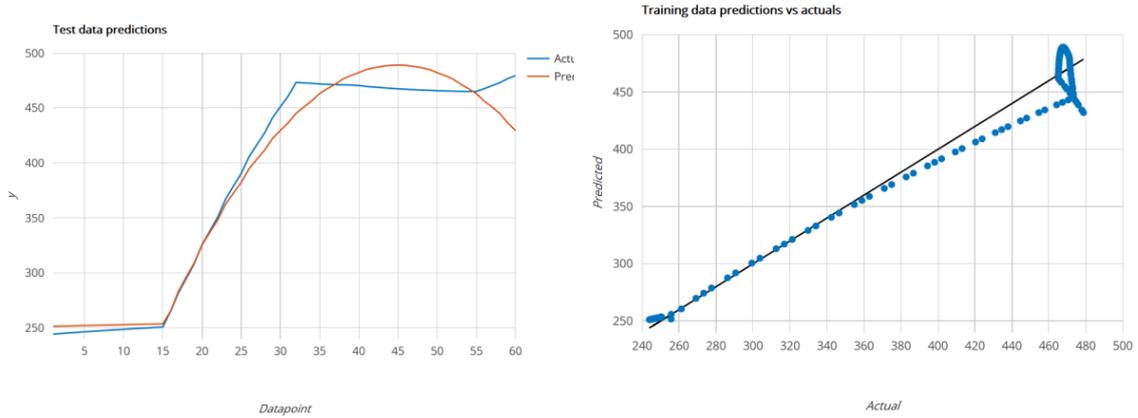


Fig 7. Test Actual with Predicted Model Graph $(\sigma_{\text{eqv}})_1$

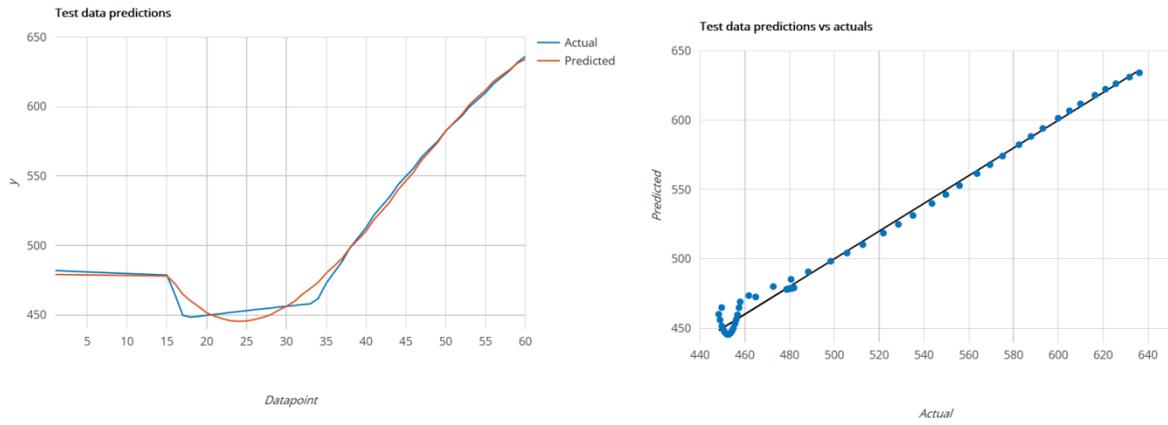


Fig 8. Test Actual with Predicted Model Graph(σ_{eqv})₂

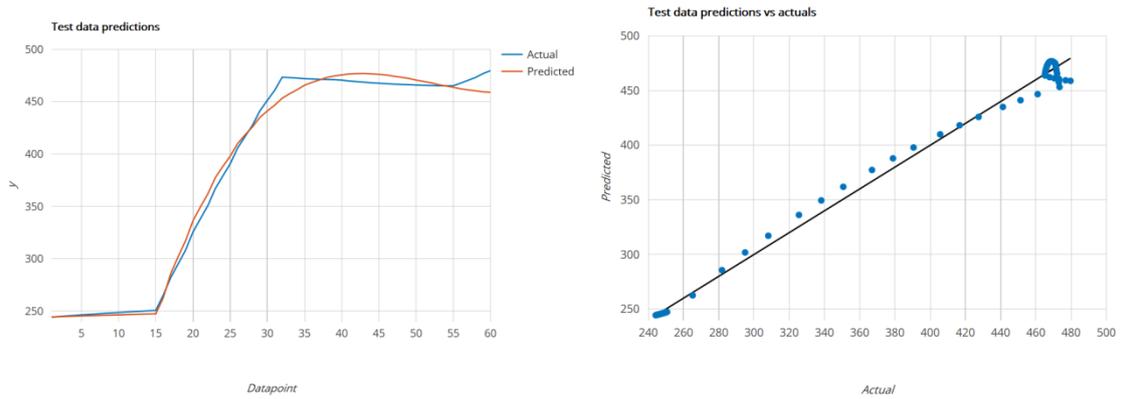


Fig 9. Test Actual with Predicted Model Graph(σ_{eqv})₃

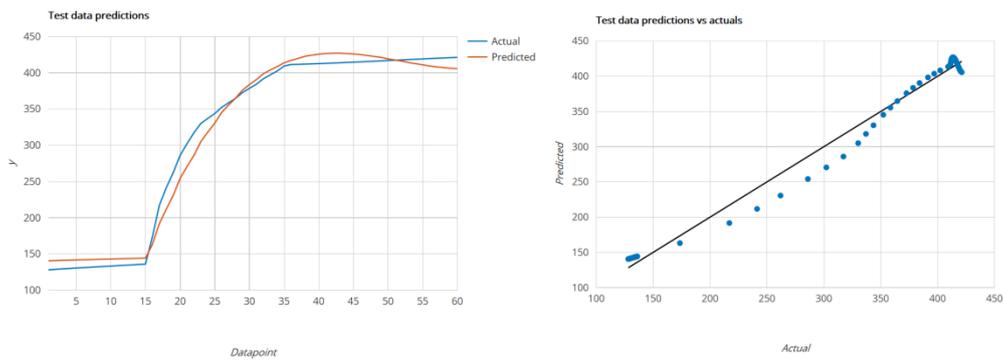


Fig 10. Test Actual with Predicted Model Graph(σ_{eqv})₄

3.2. Analysis of Evolved Models

The evolved trees information of best solutions are shown in Table 5. GP extracted successful models with few features by evaluating the data sets. Extracted best models of equations have the best performance in equivalent stress are given Table 6.

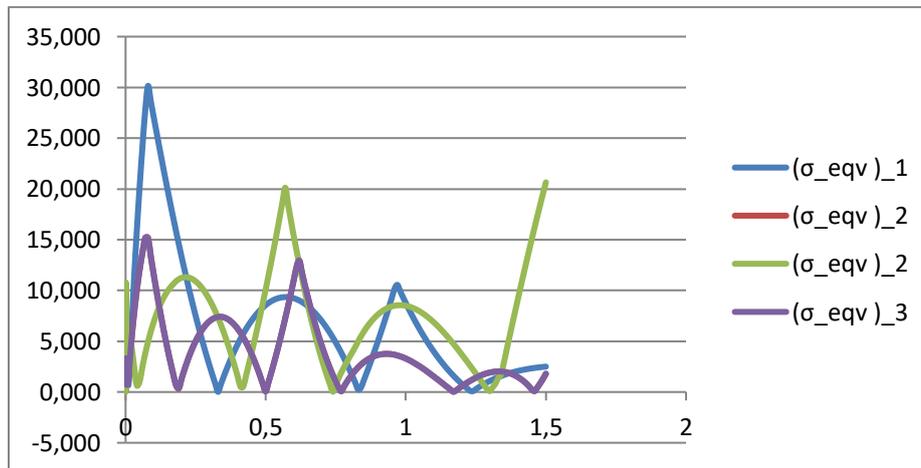
Table 5. Best solution tree information for each data set

Problem	Best Model Criteria	Metric
$(\sigma_{\text{eqv}})_1$	Node	11
	Complexity	33
	Tree Depth	4
$(\sigma_{\text{eqv}})_2$	Node	11
	Complexity	33
	Tree Depth	4
$(\sigma_{\text{eqv}})_3$	Node	13
	Complexity	41
	Tree Depth	4
$(\sigma_{\text{eqv}})_4$	Node	15
	Complexity	49
	Tree Depth	4

Table 6. Best model of equations

Dataset	Equations
$(\sigma_{\text{eqv}})_1$	$-58.57m_1^3 + 234.0 m_1^2 - 83.22m_1 + 587.1$
$(\sigma_{\text{eqv}})_2$	$-158.5m_1^3 + 457.1m_1^2 - 235.9m_1 + 479.2$
$(\sigma_{\text{eqv}})_3$	$146.5m_1^3 - 543.8m_1^2 + 629.3m_1 + 244.2$
$(\sigma_{\text{eqv}})_4$	$184.4m_1^3 - 678.5m_1^2 + 779.7m_1 + 140.4$

Numerical (real) values corresponding to 200 different composition gradient upper values (m values) were produced in the range of 0.001-1.5 of the one dimensional functionally graded rectangular plates. These values are used as datasets in GP. In this study, we obtained error values using the actual values and predicted values using Eq. (2). The graph of error values of all datasets in equivalent stresses is shown in Figure 11. As shown in Figure 11, the error values are fluctuated in all datasets.

**Fig 11. Error Distributions in All Datasets**

20 randomly selected samples are shown in Table 7. Table 7 show that the minimum error value is 0.01 and the error rate is 0.0017% ; the maximum error value is 30.14 and the error rate is 5.46%. The RMSE value is 8.53.

Table 7. Actual, GP Predicted and Error Values in $(\sigma_{\text{eqv}})_1$

m	$(\sigma_{\text{eqv}})_1$ with	$(\sigma_{\text{eqv}})_1$ with	$(\sigma_{\text{eqv}})_1$ with
	FDM	h GP	GP Error
0,0001	594,91	587,09	7,82
0,0002	594,83	587,08	7,74
0,003	592,44	586,85	5,59
0,08	551,77	581,91	30,14
0,13	557,37	580,11	22,74
0,17	562,25	579,43	17,17
0,33	583,00	583,02	0,01
0,48	602,65	594,59	8,06
0,43	596,14	589,93	6,21
0,49	603,94	595,61	8,33
0,5	605,23	596,67	8,56
0,74	635,27	629,92	5,35
1,17	714,64	716,25	1,61
1,19	719,68	720,74	1,06
1,2	722,18	722,99	0,80
1,21	724,68	725,24	0,56
1,26	737,04	736,58	0,47
1,28	741,94	741,13	0,80

Real and predicted values for $(\sigma_{\text{eqv}})_2$ were obtained by selecting the range [0.001-1.5]. 20 randomly selected samples are shown in Table 8. Table 8 show that the minimum error value is 0.02 and the error rate is 0.004% ; the maximum error value is 15,18 and the error rate is 3.4%. The RMSE value is 4.67.

Table 8. Actual, GP Predicted and Error Values in $(\sigma_{\text{eqv}})_2$

m	$(\sigma_{\text{eqv}})_2$ with	$(\sigma_{\text{eqv}})_2$ with	$(\sigma_{\text{eqv}})_2$ with
	FDM	GP	GP Error
0,0001	482,13	479,18	2,96
0,0002	482,06	479,15	2,91
0,003	480,10	478,50	1,61
0,08	447,99	463,17	15,18
0,13	448,97	455,92	6,95
0,17	449,78	451,55	1,78
0,33	453,00	445,60	7,40
0,48	455,85	454,26	1,59
0,43	454,93	450,04	4,88
0,49	456,04	455,25	0,78
0,5	456,22	456,29	0,07
0,74	490,93	492,58	1,64
1,17	582,46	582,43	0,02
1,19	586,05	586,43	0,38
1,2	587,83	588,40	0,57
1,21	589,59	590,36	0,76
1,26	598,26	599,80	1,54
1,28	601,65	603,41	1,76
1,34	611,54	613,56	2,02
1,5	636,21	634,41	1,80

Real and predicted values for $(\sigma_{\text{eqv}})_3$ were obtained by selecting the range [0.001-1.5]. 20 randomly selected samples are shown in Table 9. Table 9 show that the minimum error value is 0.02 and the error rate is 0.005%; the maximum error value is 20.65 and the error rate is 4.3%. The RMSE value is 7.6.

Table 9. Actual, GP Predicted and Error Values in $(\sigma_{\text{eqv}})_3$

m	$(\sigma_{\text{eqv}})_3$ with h FDM	$(\sigma_{\text{eqv}})_3$ with GP	$(\sigma_{\text{eqv}})_3$ with GP Error
0,0001	243,97	244,26	0,29
0,0002	244,11	244,33	0,21
0,003	248,07	246,08	1,99
0,08	286,22	291,14	4,92
0,13	308,17	317,14	8,97
0,17	325,56	336,18	10,62
0,33	390,60	397,91	7,31
0,48	444,69	437,17	7,51
0,43	427,52	425,90	1,62
0,49	448,02	439,23	8,80
0,5	451,33	441,21	10,11
0,74	471,44	471,46	0,02
1,17	465,97	470,71	4,74
1,19	465,82	469,87	4,05
1,2	465,75	469,44	3,69
1,21	465,69	469,01	3,32
1,26	465,40	466,84	1,43
1,28	465,31	465,97	0,67
1,34	465,25	463,51	1,75
1,5	479,69	459,04	20,65

Real and predicted values for $(\sigma_{\text{eqv}})_4$ were obtained by selecting the range [0.001-1.5]. 20 randomly selected samples are shown in Table 10. Table 10 show that the minimum error value is 0.17 and the error rate is 0.028% ; the maximum error value is 31.78 and the error rate is 5.65%. The RMSE value is 13.67.

Table 10. Actual, GP Predicted and Error Values in $(\sigma_{\text{eqv}})_4$

m	$(\sigma_{\text{eqv}})_4$ with FDM	GP $(\sigma_{\text{eqv}})_4$	$(\sigma_{\text{eqv}})_4$ with GP Error
0,0001	127,83	140,40	12,65
0,0002	128,00	140,40	12,56
0,003	132,65	140,40	10,09
0,08	225,68	140,40	27,15
0,13	262,00	140,40	31,30
0,17	286,03	140,40	31,78
0,33	343,71	140,40	13,27
0,48	374,63	140,40	4,09
0,43	364,71	140,40	0,17
0,49	376,57	140,40	4,67
0,5	378,50	140,40	5,18
0,74	411,70	140,40	8,85
1,17	416,90	140,40	2,29
1,19	417,16	140,40	1,00
1,2	417,30	140,40	0,34
1,21	417,45	140,40	0,33
1,26	418,12	140,40	3,62
1,28	418,38	140,40	4,90
1,34	419,18	140,40	8,61
1,5	421,26	140,40	15,59

The RMSE, minimum and maximum error values in the effective value range are shown in Table 11. RMSE values generated in GP under the boundary conditions [0.001-1.5], the best result is achieved in $(\sigma_{\text{eqv}})_2$. The minimum error value is $(\sigma_{\text{eqv}})_1$ and the maximum value is $(\sigma_{\text{eqv}})_4$.

Table 11. RMSE Minimum and Maximum Error Values in The Effective Value Range

	$(\sigma_{eqv})_1$	$(\sigma_{eqv})_2$	$(\sigma_{eqv})_3$	$(\sigma_{eqv})_4$
RMSE	8.53	4.67	7.6	13.67
Min Error	0.01	0.02	0.02	0.17
Max Error	30.14	15.18	20.65	31.78

4. Conclusions

In this study, equation sets were found by Genetic Programming (GP) for thermal stress analysis of functional grade rectangular plates. The variable ‘m’ value in the set of equations is the composition gradient upper value. In these equations, different equivalent stress levels are obtained by substituting ‘m’ value. GP provided significant time gain in deriving sets of equations for thermal stress analysis of plates with current boundary conditions. In the study, the models in the range of [0.001-1.5] have been observed more efficiently. The error rates obtained by GP are: 1.07% for $(\sigma_{eqv})_1$, 0.72% for $(\sigma_{eqv})_2$, 1.47% for $(\sigma_{eqv})_3$ and 4.59% for $(\sigma_{eqv})_4$. In the future works, it is aimed to change the parameters such as tree depth, functions used in the extracted models and compare the method with other automatic programming methods.

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