Title: Measuring the Change by Using Markov Chain Approach in Time-Dependent
Transitions and a Real Data Application

Authors: Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data Application

Nihan Potas[*1], Cemal Atakan[2]

Abstract

In this study, the use of the Markov chain to measure the change in time-dependent transitions is emphasized. Contingency tables were used to measure the time-dependent change of categorical data. Theoretically how to apply the Markov chain in the log-linear model with the help of one-step or higher-step transition matrices was demonstrated. In addition, the stationarity approach and the assessment of the order of the chain were given as the assumption of the model. In the real data application, 1217 undergraduate students, studying in Faculty of Political Science, Engineering, Science departments of Ankara University, were used. It was taken their cumulative average grades for 4 years, average grades for 8 semesters, beginning in the academic year 2013-2014.Whether the change in the success of the students is measurable in 8 semesters and 4 years, has been investigated. According to the results, before making any prediction: it concluded that one-step transition probabilities are not stationary and the three-step transition matrix is the second-order Markov Chain.

Keywords: Contingency tables; Markov Chain; Log-Linear Analysis; Multinomial Distrubution.

1. INTRODUCTION

Stochastic modeling methods for categorical data are widely used in scientific research. Several new methods have been proposed to estimate transition probabilities in discrete Markov chain and panel data analysis [6]. Stochastic modeling began with the Markov chain using contingency tables [5]. Markov chain was used to see the change in the transition from one time period to another in contingency tables. This has been accepted as the intentions of individuals to change in transition situations in many studies of social sciences [10]. Individual change in social sciences and social behavior, with some basic techniques as log-linear models has been established [9]. Not only in social science, change in time and interactions high dimensional is also explored in genomic data with the Bayesian model [8]. In log-linear models, missing data is an important problem when the transitions probabilities were calculated. Recommendations have been made for the problem of missing data in the sample transformation process due to lack of sample size [11]. Measurement of the change is achieved (in a way) by using several known different approaches. One of these approaches is the multinomial distribution of

---

[*] Corresponding Author: nihan_potas@hotmail.com

[1] AHBV University, Department of Health Care Management, Ankara, Turkey. ORCID: 0000-0002-0393-313

[2] Ankara University, Department of Statistics, Ankara, Turkey. ORCID: 0000-0001-6943-1675

Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data App...

the mass probabilities and the equality of change in these possibilities $\pi_{12} = \pi_{21}$. This is also the same as the hypothesis test for the equality of marginal probabilities for binomial distributions used in the 2×2 contingency tables. However, this approach cannot exhibit the whole social process, which is intended to be emphasized for social scientists working on change [7]. Another approach is the hypothesis testing of two ratio $\pi_{12}/\pi_{1+} = \pi_{21}/\pi_{2+}$ equations by reorganizing the contingency table. The limitation of this hypothesis is that there is no interaction term. This shows that you need other approaches.

Due to this requirement, stochastic models based on Markov chains were introduced and applied with time-dependent data. Markov chain data on the contingency table is shown. Moreover, how to analyze and synchronize these data to the log-linear model, which is a contingency table analysis, is demonstrated [4].

In the second part of the study, definitions and notations and the third part of the study, the log-linear model is introduced. In the fourth part, the log-linear model is related to first-order Markov models and the transition probabilities stationary, which is one of the first-order model assumptions, and the estimation of transition probabilities in first-order Markov models are emphasized. In the fifth section, the log-linear model is associated with the higher-order Markov models, the estimation of the probability of transition in higher-order Markov models and the determination of the order of the chain, which is the assumption of higher-order Markov models.

In the sixth part, 1217 undergraduate students, studying in Faculty of Political Science, Engineering, Science departments of Ankara University, were used. It was taken their cumulative average grades for 4 years, average grades for 8 semesters, beginning in the academic year 2013-2014. Whether the change in the success of the students is measurable in 8 semesters and 4 years, has been investigated. In the last section, the results of the study were evaluated and suggestions were made based on the results obtained.

2. DEFINITIONS AND NOTATION

Let Y_t, $t = 1, \dots, T$ be a random variable with a multinomial distribution of parameters π_{ij}, $i, j = 1, 2, \dots, n$. The joint probability of observations that fall to the i-th level of variable Y_{t-1} and the j-th level of variable Y_t are in the form of $P(Y_{t-1} = i, Y_t = j) = \pi_{ij}$

for $i, j = 1, 2$. The representation of these probabilities in the 2×2 contingency table is given in Table 1.

Table 1. 2×2 Cell Probabilities in Contingency Tables

	Level of Y_t		
Level of Y_{t-1}	i	j	Total
i	π_{11}	π_{12}	π_{1+}
j	π_{21}	π_{22}	π_{2+}
Total	π_{+1}	π_{+2}	1

3. LOG-LINEAR MODEL

The log-linear model of cell probabilities π_{ij} is in form

$$\log \pi_{ij} = \mu + \lambda_i^{t-1} + \lambda_j^t + \lambda_{ij}^{(t-1)t} \quad i, j = 1, 2$$

[1]. Here, the indices λ with the discrete time in t representations show $\log Y_t$ according to i, j-th levels. According to this;

$$\sum_{i=1}^{2} \lambda_i^{t-1} = \sum_{j=1}^{2} \lambda_j^t = \sum_{i=1}^{2} \lambda_{ij}^{(t-1)t} = \sum_{j=1}^{2} \lambda_{ij}^{(t-1)t} = 0.$$

Here, in case of exact symmetry is $(\pi_{12} = \pi_{21})$,

$$\lambda_i^{t-1} = \lambda_i^t \quad i = 1, 2$$

and in case of relative symmetry is $(\pi_{12}\pi_{22} = \pi_{21}\pi_{11})$,

$$\lambda_i^t = 0 \quad i = 1, 2.$$

There is no need to fix line totals for the relative symmetry test. If the sample size is constant, then both exact symmetry and relative symmetry models can be considered. On the other hand, if the line totals are fixed, the relative symmetry model should be employed. For this, McNemar and Bowker tests are used for exact symmetry and Yates's corrected chi-square test is used for relative symmetry [12].

Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data App...

4. FİRST-ORDER MARKOV MODELS

In order to measure the change, it is necessary to redefine the cell probabilities so that row sum is 1. Here, the first sub-indices show the initial state, the second sub-index shows the second state. In this case, Table 1, which we have given previously, should be rewritten as Table 2.

Table 2. 2×2 Transition Probabilities in Contingency Tables

	Level of Y_t		
Level of Y_{t-1}	i	j	Total
i	$P_{11} = \dfrac{\pi_{11}}{\pi_{1+}}$	$P_{12} = \dfrac{\pi_{12}}{\pi_{1+}}$	1
j	$P_{21} = \dfrac{\pi_{21}}{\pi_{2+}}$	$P_{22} = \dfrac{\pi_{22}}{\pi_{2+}}$	1

With this new parametrization, the transition probabilities expressed as P_{ij} indicate $i \rightarrow j$ or $j \rightarrow i$ transition levels and whether or not they remain at the same level. According to Table 2, transition probabilities showed that, the probabilities of staying at the same-level are (P_{11}, P_{22}), the probabilities of making the transition from another-level are (P_{21}, P_{12}).

4.1. Approach to Stationary

When looking at time-dependent observations, it is expected that the structure will be preserved in transitions between the times. The most important reason for this is to find the expected ratios at the time of $(t + 1)$. In order to obtain the patterns of the expected ratios, $R_{i+}(t)$ and $R_{j+}(t)$, which are the ratios of the i-th and j-th levels in time t, must be known.

The relation between these ratios is expressed as $R_{i+}(t) + R_{j+}(t) = 1$. The expected ratios for time $t + 1$ are also indicated by

$$R_{i+}(t + 1) = R_{i+}(t)P_{11} + R_{j+}(t)P_{21}$$

$$R_{j+}(t + 1) = R_{i+}(t)P_{12} + R_{j+}(t)P_{22}.$$

Here, $R_{i+}(t)P_{11}$ and $R_{j+}(t)P_{22}$ are at the initial level and the proportion of individuals who remain at the same level; $R_{j+}(t)P_{21}$ and $R_{i+}(t)P_{12}$ are at the initial

level and show the ratio of those who pass to different levels.

$R_{i+}(t + 1) = R_{i+}(t)P_{11} + R_{j+}(t)P_{21}$ equality is the ratio of the i-th level at time $(t + 1)$.

Here, $R_{i+}(t)P_{11}$ shows the proportion of individuals at i-th level initially and still at the i-th level and $R_{j+}(t)P_{21}$ shows the proportion of individuals at the j-th level initially and pass the i-th level

$R_{j+}(t + 1) = R_{i+}(t)P_{12} + R_{j+}(t)P_{22}$ equality is the ratio of the j-th level at time $(t + 1)$.

Here, $R_{i+}(t)P_{12}$ shows the proportion of individuals at the i-th level initially and pass the j-th level and $R_{j+}(t)P_{22}$ shows the proportion of individuals at the j-th level initially and still at the j-th level.

Under the assumption of stationary, $P_{ij} \; i, j = 1,2$ is the probability of transition of the levels, and the process of these transition possibilities is

$$R_{i+}(t + 1) = R_{i+}(t)P_{11} + R_{j+}(t)P_{21}$$

$$= R_{i+}(t)(1 - P_{12}) + R_{j+}(t)P_{21}$$

$$= P_{21} + (1 - P_{12} - P_{21})R_{i+}(t).$$

When it is assumed that $P_{21}(t) + P_{12}(t) \neq 0$ and $\alpha = 1 - P_{12}(t) - P_{21}(t)$, and the initial probabilities are $R_{1+}(1), R_{2+}(1)$, then we have

$$R_{1+}(2) = P_{21} + \alpha R_{1+}(1)$$

$$R_{1+}(3) = P_{21} + \alpha R_{1+}(2)$$

$$= P_{21} + \alpha(P_{21} + \alpha R_{1+}(1))$$

$$= P_{21} + \alpha P_{21} + \alpha^2 R_{1+}(1)$$

$$R_{1+}(4) = P_{21}(1 + \alpha + \alpha^2) + \alpha^3 R_{1+}(1)$$

$$\vdots$$

$$R_{1+}(t) = P_{21}(1 + \cdots + \alpha^{t-2}) + \alpha^{t-1}R_{1+}(1)$$

$$= \frac{P_{21}}{P_{21} + P_{12}}(1 - \alpha^{t-1}) + \alpha^{t-1}R_{1+}(1).$$

Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data App...

Because of $P_{21}(t) + P_{12}(t) \neq 0$, when t goes to infinity α^{t-1} is approaches zero. Therefore when the limit of the probabilities is taken, then

$$\lim_{t \to \infty} R_{1+}(t) \approx \frac{P_{21}}{P_{21} + P_{12}}$$

as a result of limit, it is approaches

$$R_{1+}(t) \longrightarrow \frac{P_{21}}{P_{21} + P_{12}}.$$

This shows that the transition probabilities in the process are very slow and small at every time interval [3]. Another important result is that the limit values are independent from the initial probability. However, if any $R_{1+}(t)$ is equal to this limit value, then $R_{1+}(t+1)$ is equal.

4.2. Estimation of transition probabilities in first-order Markov Models

Let it be defined as $y_{ij}(t)$ is the number of individuals at the i-th level at time $(t-1)$ and at the j-th level at time t, $y_i(t-1)$ is the number of individuals at the i-th level at time $(t-1)$, $y_j(t)$ is the number of individuals at the j-th level at time t. It can be written as

$$y_{+j}(t) = y_j(t)$$

$$y_{i+}(t) = y_i(t-1)$$

and total number of individuals $n = \sum_{i=1}^{I} \sum_{j=1}^{J} y_{ij}(t)$ by using $y_{i+}(t) = y_i(t-1)$. Let I be the number of possible levels and let T be observation set of transitions (first transition $0 \to 1$). $y_i(0)$ is the number of individuals at i-th level at time $t = 0$ and the distribution of $\{y_i(0)\} \sim Multinomial(\eta_i)$. The sample size can be defined as $n = \sum_{i=1}^{I} y_i(0)$. Let $P_{ij}(t)$, be the conditional probability of being at the j-th level at time t, when given the i-th level at time $(t-1)$. Therefore, for each i and t is $\sum_{j=1}^{I} P_{ij}(t) = 1$. When $y_{i+}(t) = y_i(t-1)$ is given and i is fixed, conditional distribution of $y_{ij}(t)$ for $j = 1, ..., I$ is

$$\frac{y_i(t-1)!}{\prod_{j=1}^{I} y_{ij}(t)!} \prod_{j=1}^{I} P_{ij}(t)^{y_{ij}(t)}.$$

If all transitions are mutually independent, the joint probability distribution $y_{ij}(t)$ and $y_i(0)$ is written as

$$\left[\frac{n!}{\prod_{i=1}^{I} y_i(0)!} \prod_{i=1}^{I} \eta_i^{y_i(0)} \right]$$
$$\times \prod_{t=1}^{T} \left\{ \prod_{i=1}^{I} \left[\frac{y_i(t-1)!}{\prod_{j=1}^{I} y_{ij}(t)!} \prod_{j=1}^{I} P_{ij}(t)^{y_{ij}(t)} \right] \right\}.$$

Where η_i, $P_{ij}(t)$ represent the probability of occurrence of $y_i(0), y_{ij}(t)$. The maximum likelihood function of η_i, $P_{ij}(t)$ is

$$\left[\prod_{i=1}^{I} \eta_i^{y_i(0)} \right] \times \left[\prod_{t=1}^{T} \prod_{i=1}^{I} \prod_{j=1}^{I} P_{ij}(t)^{y_{ij}(t)} \right]$$

and maximum likelihood estimates for parameters are

$$\hat{\eta}_i = \frac{y_i(0)}{n}, \hat{P}_{ij}(t) = \frac{y_{ij}(t)}{y_i(t-1)} = \frac{y_{ij}(t)}{y_{i+}(t)}.$$

If the Markov chain has stationary transition probabilities, it is is written as $P_{ij}(t) = P_{ij}$ for $t = 1, ..., T$. In this case the likelihood function can be written as

$$\left[\prod_{i=1}^{I} \eta_i^{y_i(0)} \right] \times \left[\prod_{i=1}^{I} \prod_{j=1}^{I} P_{ij}^{y_{ij}} \right].$$

If it is expressed as $y_{ij} = \sum_{t=1}^{T} y_{ij}(t)$, the maximum likelihood estimates for the parameters in the stationary transition possibilities are

$$\hat{\eta}_i = \frac{y_i(0)}{n}, \hat{P}_{ij}(t) = \frac{y_{ij}(t)}{y_{i+}(t)} = \frac{\sum_{t=1}^{T} y_{ij}(t)}{\sum_{t=1}^{T} y_i(t-1)}.$$

In the first-order Markov chain, the observed transition matrices are compared in order to check the stationary of the transition probabilities. These matrices form $I \times I \times T$ contingency table. The stationary of the transition probabilities corresponds to the log-linear model of

$$\log \pi_{ijk} = \mu + \lambda_i^{t-2} + \lambda_j^{t-1} + \lambda_k^t \qquad (1)$$
$$+ \lambda_{ij}^{(t-2)(t-1)} + \lambda_{ik}^{(t-2)t}$$

Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data App...

for expected cell frequencies. With this model, the level at the initial of the transition is given, and it should be checked whether the level at the end of the transition is independent of the transition time [2].

5. HIGHER-ORDER MARKOV MODELS

In the first-order Markov models, the order of the transitions between the set of points I is examined at T fixed point in time and the process of change is described as dependent on one-step transitions. This is shown as transition from $(t-1)$ to t. Generally, the situations encountered are different. The occurrence of the current level may not be limited to two times, but may be due to different time levels. For this, the period of time $t-1, t-2, ..., t-r$ as $r > 1$ can be re-defined. In this case it means that r-th order Markov chain.

5.1. Estimation of transition probabilities in higher-order Markov models

The second-order Markov models with the same characteristics will be studied in order to more easily explain the k-th order Markov chains $(k < r)$. $y_i(0)$ is the number of individuals at the i-th level at time $t = 0$ and $\{y_i(0)\} \sim Multinomial(\eta_i)$. The sample size is $n = \sum_{i=1}^{I} y_i(0)$.

Let $y_{ijk}(t)$ be the number of individuals at the level of i at the time of $(t-2)$, at the level of j at the time of $(t-1)$ and at the level of k at the time of t; let $y_{ij}(t-1)$ be the number of individuals at the level of i at the time of $(t-2)$, at the level of j at the time at $(t-1)$.

Let $y_{jk}(t)$ be the number of individuals at the level of j at the time of $(t-1)$ and at the level of k at time of t; let $y_i(t-2)$ be the number of individuals at the level of i at the time of $(t-2)$.

Let $y_j(t-1)$ be the number of individuals at the level of j at the time of $(t-1)$; let $y_k(t)$ be the number of individuals at the level of k at the time of t. For $t = 2, ..., T$;

$$y_{ij+}(t) = y_{ij}(t-1)$$

$$y_{+jk}(t) = y_{jk}(t)$$

$$y_{i++}(t) = y_i(t-2)$$

$$y_{+j+}(t) = y_j(t-1)$$

$$y_{++k}(t) = y_k(t)$$

can be written and total number of individuals are $n = \sum_{i,j,k} y_{ijk}(t)$.

If the two-step transitions are mutually independent, the joint probability distribution of $y_{ijk}(t)$, $y_{ij}(1)$ and $y_i(0)$ is written as

$$\left[\frac{n!}{\prod_{i=1}^{I} y_i(0)!} \prod_{i=1}^{I} \eta_i^{y_i(0)} \right]$$
$$\times \prod_{i=1}^{I} \left[\frac{y_i(0)!}{\prod_{j=1}^{I} y_{ij}(1)!} \prod_{j=1}^{I} \xi_{ij}^{y_{ij}(1)} \right]$$
$$\times \prod_{t=2}^{T} \left\{ \prod_{i=1}^{I} \prod_{j=1}^{I} \left[\frac{y_{ij}(t-1)!}{\prod_{j=1}^{I} y_{ijk}(t)!} \prod_{k=1}^{I} P_{ijk}(t)^{y_{ijk}(t)} \right] \right\}.$$

Here, η_i, ξ_{ij} and $P_{ijk}(t)$ represent the probability of occurrence of $y_i(0), y_{ij}(1)$ and $y_{ijk}(t)$. From here,

$$\sum_j \xi_{ij} = 1 \quad i = 1, ..., I$$

$$\sum_k P_{ijk}(t) = 1 \quad i, j = 1, ..., I; t = 1, ..., T.$$

the likelihood function of η_i, ξ_{ij} and $P_{ijk}(t)$ is obtained as

$$\left[\prod_{i=1}^{I} \eta_i^{y_i(0)} \right] \times \left[\prod_{i=1}^{I} \prod_{j=1}^{I} \xi_{ij}^{y_{ij}(1)} \right]$$
$$\times \left[\prod_{t=2}^{T} \prod_{i=1}^{I} \prod_{j=1}^{I} \prod_{k=1}^{I} P_{ijk}(t)^{y_{ijk}(t)} \right]$$

and the maximum likelihood estimation of the parameters is

$$\hat{\eta}_i = \frac{y_i(0)}{n}, \hat{\xi}_{ij} = \frac{y_{ij}(1)}{y_i(0)}, \hat{P}_{ijk}(t) = \frac{y_{ijk}(t)}{y_{ij}(t-1)}.$$

If the second-order chain has stationary transition possibilities matrix, then it becomes $P_{ijk}(t) = P_{ijk}$ for $t = 2, ..., T$. In this case the likelihood function is

Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data App...

$$\left[\prod_{i=1}^{I} \eta_i{}^{y_i(0)}\right] \times \left[\prod_{i=1}^{I}\prod_{j=1}^{I} \xi_{ij}^{y_{ij}(1)}\right]$$
$$\times \left[\prod_{i=1}^{I}\prod_{j=1}^{I}\prod_{k=1}^{I} P_{ijk}{}^{y_{ijk}}\right].$$

If $y_{ijk} = \sum_{t=2}^{T} y_{ij}(t)$ and $\sum_{k=1}^{I} y_{ijk}(t) \neq y_{ij}$, the maximum likelihood estimate of P_{ijk} is

$$\hat{P}_{ijk} = \frac{y_{ijk}}{\sum_{k=1}^{I} y_{ijk}}.$$

In order to check the stationary of the transition probabilities in the second-order Markov chain, the term $\lambda^{(t-1)t}$ of the log-linear model and the set of higher-order terms of λ that are associated with it are equal to zero. For the model

$$\log \pi_{ijkl} \qquad (2)$$
$$= \mu + \lambda_i^{t-3} + \lambda_j^{t-2} + \lambda_k^{t-1} + \lambda_l^t + \lambda_{ik}^{(t-3)(t-1)}$$
$$+ \lambda_{jk}^{(t-2)(t-1)} + \lambda_{il}^{(t-3)t} + \lambda_{ij}^{(t-3)(t-2)}$$
$$+ \lambda_{jl}^{(t-2)t} + \lambda_{ijk}^{(t-3)(t-2)(t-1)} + \lambda_{ijl}^{(t-3)(t-2)t}$$

calculation of the $I \times I \times I \times T$ contingency table is $y_{ijkl} = y_{ijkl}(t)$. To test stationarity, The chi-square goodness-fit statistic with $IJ(K-1)(L-1)$ degrees of freedom is used. The test statistics are

$$\chi^2 = \sum_{ijkl} \frac{\left(y_{ijkl} - y_{ij+l}y_{ijk+}/y_{ij++}\right)^2}{y_{ij+l}y_{ijk+}/y_{ij++}}$$

and

$$G^2 = 2\sum_{ijkl} y_{ijkl} \ln\left(\frac{y_{ijkl}}{y_{ij+l}y_{ijk+}/y_{ij++}}\right)$$

[4].

5.2. Determination of the order of the chain

Assume that there is a first-order or second-order Markov chain with the possibility of a stationary transition. In the next step, the calculation of the $I \times I \times I$ contingency table is

$$y_{ijk+} = \sum_{t=2}^{T} y_{ijk}(t).$$

Saturated general log-linear model and if there is a first-order chain, the model is defined as

$$\log \pi_{ijk} = \mu + \lambda_i^{t-2} + \lambda_j^{t-1} + \lambda_k^t \qquad (3)$$
$$+ \lambda_{ij}^{(t-2)(t-1)} + \lambda_{jk}^{(t-1)t}.$$

Thus, the second-order chain tested becomes the first-order chain. Log-linear model is used to test the standard goodness of fit test statistic.

Assuming that the chain here is a stationary transition probability, it determines whether the chain is first or second order. If the transition probabilities are stable, predicted transition probabilities could be written by combining tables. This makes the hypothesis equivalent to the first-order Markov chain. In the following section, the application of real data will be explained and the theoretical information will be reinforced with detailed information about the analysis of the models.

6. APPLICATION

The average grades for 8 semesters of 1217 undergraduate students, studying in Faculty of Political Science, Engineering, Science departments of Ankara University, beginning in the academic year 2013-2014, were used. Students' both the cumulative-average grades and the average grades were requested from Ankara University, student affairs office without their personal information. One-step transition matrices according to academic achievement grade point average (YABNO) values are given in Table 3.-9. The success level of the students in the tables, those with a grade point average of 2.00 and above were regarded as successful. H_0 is examined whether the one-step transition probabilities are stationary (H_0: Transition probabilities are stationary).

For the first-order Markov model, the goodness of fit test results were found to be $\chi^2 = 276.081$, $p < 0.05$ and $G^2 = 273.391$, $p < 0.05$ with 12 degrees of freedom. The null hypothesisis established on the basis that the transition probabilities are stationary. In this case, it was concluded that the transition probabilities were not stationary.

Table 3. 2 × 2 Contingency Table for 2013-2014 Fall and Spring Semester

		13_14 Spring		Total
		(a)	(b)	
13_14	(a)	429	48	477
Fall	(b)	158	582	740
Total		587	630	1217

(a) Unsuccesful (b) Successful

Table 4. 2 × 2 Contingency Table for 2013-2014 Spring and 2014-2015 Fall Semester

		14_15 Fall		Total
		(a)	(b)	
13_14	(a)	492	95	587
Spring	(b)	168	462	630
Total		660	557	1217

(a) Unsuccesful (b) Successful

According to the log-linear model in Equation 2; Table 3. and Table 4. transition probabilities matrices were tested by using goodness of fit test with 2 degrees of freedom. The results are $\chi^2=13.777$, $p<0.05$ and $G^2=13.948$, $p<0.05$. It is concluded that transition possibilities are not stationary.

Table 5. 2 × 2 Contingency Table for 2014-2015 Fall and Spring Semester

		14_15 Spring		Total
		(a)	(b)	
14_15	(a)	585	75	660
Fall	(b)	133	424	557
Total		718	499	1217

(a) Unsuccesful (b) Successful

According to the log-linear model in Equation 3; Table 3., Table 4. and Table 5. transition matrices were

examined by using the goodness of fit test with 4 degrees of freedom. The Pearson chi-square is $\chi^2=15.816$, $p<0.05$ and the likelihood ratio is $G^2=15.558$, $p<0.05$ and transition possibilities are not stationary.

Table 6. 2 × 2 Contingency Table for 2014-2015 Spring and 2015-2016 Fall Semester

		15_16 Fall		Total
		(a)	(b)	
14_15	(a)	572	146	718
Spring	(b)	69	430	499
Total		641	576	1217

(a) Unsuccesful (b) Successful

In Table 3., Table 4., Table 5. and Table 6. the transition matrices were used to determine whether it was stationary or not. $\chi^2=61.673$, $p<0.05$ and $G^2=63.266$, $p<0.05$ with 6 degrees of freedom showed that transition possibilities are not stationary. In Table 6., Table 7., Table 8. and Table 9. transition probabilities matrices was tested by using goodness of fit test according to model. The results are $\chi^2=133.717$, $p<0.05$ and $G^2=127.054$, $p<0.05$ with 6 degrees of freedom. it was observed that the transition probabilities were not stationary.

Table 7. 2 × 2 Contingency Table for 2015-2016 Fall and Spring Semester

		15_16 Spring		Total
		(a)	(b)	
15_16	(a)	544	97	641
Fall	(b)	131	445	576
Total		675	542	1217

(a) Unsuccesful (b) Successful

In Table 7., Table 8. and Table 9. transition probabilities matrices were investigated with 4 degrees of freedom and the results showed that $\chi^2=131.430$, $p<0.05$ and $G^2=125.943$, $p<0.05$, transition probabilities were not stationary.

Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data App...

Table 8. 2×2 Contingency Table for 2015-2016 Spring and 2016-2017 Fall Semester

		16_17 Fall		Total
		(a)	(b)	
15_16	(a)	452	223	675
Spring	(b)	49	493	542
Total		501	716	1217

(a) Unsuccesful (b) Successful

Table 9. 2×2 Contingency Table for 2016-2017 Fall and Spring Semester

		16_17 Spring		Total
		(a)	(b)	
16_17	(a)	417	84	501
Fall	(b)	71	645	716
Total		488	729	1217

(a) Unsuccesful (b) Successful

According to the log-linear model in Equation 2; in Table 8. and Table 9. transition probabilities matrices were tested with 2 degrees of freedom and were obtained as χ^2=39.738, p <0.05 and G^2=41.129, p <0.05. The transition probabilities are not stationary.

The first two, three, four one-step transition matrices were compared to the last two, three and four one-step transition matrices. As a result of this comparison, it has been observed from the results that it does not disrupt the similar characteristics, that is, the transition probabilities do not provide stationary.

The cumulative-average grades for 4 years of 1217 undergraduate students, studying in Faculty of Political Science, Engineering, Science departments of Ankara University, beginning in the academic year 2013-2014, were used. One-step transition matrices according to academic achievement grade point average (GABNO) values are given in Table 10. The success level of the students in Table 10. is determined according to 2.00 grade point average. This real data application was assumed to be the first-order Markov chain under the assumption that the level of change was still in progress. Here, it was determined whether the chain

was first-level or second-level under the assumption that the chain probabilities was stationary when there was no stationary. The three-step transitions matrice is given in Table 10.

Table 10. $2 \times 2 \times 2 \times 2$ Contingency Table for 2013-2017 Years

Year 13_14	Year 14_15	Year 15_16	Year 16_17 (a)	Year 16_17 (b)	Total
(a)	(a)	(a)	244	95	339
		(b)	9	83	92
	(b)	(a)	6	5	11
		(b)	1	82	83
(b)	(a)	(a)	35	8	43
		(b)	4	45	49
	(b)	(a)	7	11	18
		(b)	3	579	582

(a) Unsuccesful (b) Successful

The hypothesis here is the same as the first-order Markov chain hypothesis. The transition matrices in Table 10. were investigated by using goodness of fit test with 4 degrees of freedom, according to Equation 3. χ^2=362.722, p <0.05 and G^2=243.560, p <0.05 are the results and it showed that the probabilities of transition are not stationary. Under the assumption that there are transition probabilities of stationary, the chain is examined whether it is the first or second-order chain. The null hypothesisis determined as the first-order Markov chain. According to the hypothesis, by using the log-linear model in Equation 4, results of the goodness of fit test with 2 degrees of freedom are χ^2=52.277, p <0.05 and G^2=38.620, p <0.05. This means that the null hypothesisis rejected.

As a two-step, the null hypothesis should be re-examined as a second-order Markov chain. According to this, test results are χ^2=3.075, p >0.05 and G^2=3.085, p >0.05 with 4 degrees of freedom. The null hypothesis cannot be rejected. If the stationary assumption had been achieved, it could be assumed that it was a second-order Markov chain. It could make a prediction by using the transition probabilities with the appropriate model.

7. CONCLUSION AND RECOMMENDATIONS

The study is based on the theoretical application of the Markov chain in the log-linear model. In many studies, especially in the social sciences, the lack of diversity in categorical data analysis has been identified as working

Nihan Potas, Cemal Atakan

Measuring the Change by Using Markov Chain Approach in Time-Dependent Transitions and a Real Data App...

according to needs. Today, many disciplines continue to work on real data. Due to the structure of the work, it is not always possible to work with contintious data types. In the categorical data, the analyzes used generally have the same style, and the course of the research changes with the help of a vicious cycle. As a result, the basic questions of the research cannot be answered and the statistical information is lost.

Unlike the classical chi-square analysis method, log-linear analysis is applied to see the interaction between categories in contingency tables. With this method, when the time variable is included in contingency tables, it is possible to examine how much of the variable is required to be examined at a certain time with the help of Markov processes. However, in such linear models, when conditions such as the assumption of stationary are provided within the study, it is possible to have an idea by using the transition probabilities for the next period to be in proportion.

The assumptions in these models are that the probabilities of transition are stationary, so that the similar probabilities is to continue in the next time period. Then, we need to examine whether this similarity depends on how many periods before. Although the stationary of the real data part of the study was not achieved, the order of the chain was examined under the assumption that it was provided. What can be done for the use of transition probabilities for the situations in which stationary is not achieved is determined as a separate study topic. However, this method has allowed the analysis of past periods of information by including time in categorical data analysis. As a result, when assumptions are made in the models, it is certain that it will be of great benefit to learn about future periods.

References

[1]   A. Agresti, Categorical Data Analysis, Third Edition. John Wiley&Sons, Inc., Hoboken, New Jersey, 2012.

[2]   T.W. Anderson, Probability Model For Analyzing Time Changes in Attitudes.ss.17-66. P.F. Lazarsfeld, ed. 1954. In Mathematical Thinking in Social Science, Glencoe, III., The Free Pres., 1954.

[3]   T.W. Anderson, L. A., Goodman, Statistical Inference about Markov Chains. Ann. Math. Statistics, 28, 89-110, 1957.

[4]   M. M. Y. Bishop, E. S. Fienberg, W. P. Holland, Discrete Multivariate Analysis Theory and Practice. Springer, New York, 1974.

[5]   R. R. Bush, F. Mosteller, Stochastic Models for Learning. John Wiley&Sons, Inc., Hoboken, New Jersey, 1955.

[6]   F. Eskandar, M. R. Meslikani, Empirical Bayes analysis of log-linear models for a generalized finite stationary Markov chain. Metrika, 59, 173-191, 2004.

[7]   L. A. Goodman, Statistical Methods for Analyzing Processes of Change. Amer. J. Sociol., 68, 57-78, 1962.

[8]   J. Hu, A. Joshi, V.E. Johnson, Log-Linear Models for Gene Association. J. Am. Stat Assoc., 104(486),597-607, 2009.

[9]   D. Knoke, P. J. Burke, Log-Linear Models. Sage Publications, Newbury Park, CA, 1980.

[10]   A. Madansky, Test of Homogeneity for Correlated Samples. Jour. American Statist. Assoc., 58, 97-119, 1963.

[11]   J. K. Vermunt, Log-linear Models for Event Histories., Sage, Thousand Oaks, CA, 1997.

[12]   A. von Eye, C. Spiel, Standart and nonstandard Log-Linear Symmetry Models for Measuring Change in Categorical Variables, The American Statistician, 50(4), 300-305, 1996.