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# Detection of Antipodal Signals Under Skewed Alpha-Stable Noise

# Asimetrik Alfa-Kararlı Gürültü Altında Zıt Kutuplu İşaretlerin Tespiti

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## Abstract

This study analyses performance of digital communication system having baseband binary phase shift keying modulation under asymmetric non-Gaussian noise. Correspondingly, the problem is characterised as antipodal signal detection where skewed α −stable distribution is used as asymmetric non-Gaussian noise exhibiting impulsive behaviour. The performance of the communication system is expressed as a function of error probability with respect to noise parameters. It is shown that the skewness of the channel noise results in degradation of error probability at low signal to noise ratio. Also, the effect of skewness is more apparent when the impulsiveness of the noise increased. As the contribution, any receiver design to compensate the asymmetric behaviour of the channel noise can be used to enhance the detection accuracy since the error performance gets worse while departing from symmetric behaviour of the noise under the same impulsiveness. Additionally, the detection performance is analysed in terms of receiver operating characteristic under the skewed α −stable noise environment. According to findings, probability of detection can be tuned and locally increased with respect to the false alarm probability depending on the sign of the skewness.

***Keyword:*** *Skewed alpha-stable noise, signal detection,. probability of error*

## Öz

Bu çalışma, asimetrik Gauss olmayan gürültü altında temel band ikili faz kaydırmalı anahtarlamaya sahip sayısal haberleşme sisteminin başarımını analiz etmektedir. Buna karşılık olarak, problem asimetrik $\alpha$ −kararlı gürültünün asimetrik Gauss olmayan gürültü olarak kullanıldığı zıt kutuplu işaret tespiti olarak karekterize edilmektedir. Haberleşme sisteminin başarımı hata olasılığının gürültü parametreleri cinsinden fonksiyonu olarak ifade edilmektedir. Düşük işaret gürültü oranında kanal gürültüsündeki asimetrinin hata olasılığında kötüleşmeyle sonuçlandığı gösterilmektedir. Ek olarak, asimetrinin etkisi kanal gürültüsünün dürtüselliği artırıldığında daha görünür olmaktadır. Katkı olarak, kanal gürültüsünün asimetrik davranışını telafi edecek herhangi bir alıcı tasarımı, hata olasılığının aynı dürtüsellikte iken simetrik davranıştan uzaklaştıkça kötüleşmesinden dolayı tespit doğruluğunu iyileştirmede kullanılabilir. Buna ek olarak, tespit başarımı asimetrik $\alpha$ −kararlı gürültü ortamında alıcı işletim karakteristiği ile analiz edilmektedir. Bulgulara göre, tespit olasılığı asimetrinin işaretine bağlı olan yanlış alarm olasılığı ile ayarlanıp lokal olarak artırılabilir.

***Anahtar Kelimeler :*** *Asimetrik alfa-kararlı gürültü, işaret tespiti, hata olasılığı*

## 1. Introduction

In most of the statistical signal processing applications based on digital communication, the additive noise is generally modelled as white Gaussian noise. However, it is proposed in [1] that the noise may exhibit non-Gaussian impulsive behaviour which is reported to have symmetric α-stable (SαS) distribution. Deterministic signal detection problem and developing detectors in SαS noise are described and analysed in [2,3]. Subsequently, the probability of error is reported by [4] to be degraded with respect to increasing impulsiveness in binary signal classification problem. To improve the receiver performance, several detectors are introduced. In [5], the signal detection is performed via correntropy function under Gaussian mixture or SαS distributed noise. Another study utilizes suboptimal detectors such as linear, soft limiter and Cauchy detectors to determine error probability under SαS noise [6]. Alternatively, the elimination of the impulsive channel noise at the receiver was realized by proposing suboptimal detectors such as myriad, Cauchy and soft limiter to detect antipodal signals under SαS noise [7]. Rather than modelling the channel noise by only SαS distribution, it is formulated as mixture of both Gaussian and SαS noise in digital communication where the likelihood estimation is proposed via an approximate analytical density function of SαS noise [8]. Another similar approach given by [9] is to design a receiver in presence of both Gaussian and SαS distributed noise using maximum likelihood detector. A comprehensive study on detection of antipodal symbols in both SαS and Gaussian noise interference is given in [10] proposing a receiver and compares the performance with other conventional detector types such as linear, Cauchy, maximum likelihood and soft limiter detectors. Instead of baseband information bearing signals, the receiver design and the probability of error performance of several digital communication systems including different modulation types under SαS noise are given together with approximate analytical expression in [11] and more comprehensively in [12]. In a more recent study, probability density function (PDF) approach is used to detect the statistically dependent symmetric heavy-tailed signals [13].

The common assumption contained by the literature is that the α −stable distribution representing the channel noise is considered to be symmetric. However, this assumption is restrictive and the noise may practically exhibit asymmetric behaviour where the detectors under symmetric channel noise may yield misleading results for this case. Therefore, the error probability should be investigated by taking into account the noise characterization for both impulsive and asymmetric cases.

Differing from the previous studies, this paper extends the analysis of signal detection problem involving skewed α −stable distribution in order to extract the effect of asymmetric behaviour of impulsive channel noise. By constructing the binary hypothesis testing problem under the proposed noise model, the error probability is formulated and expressed analytically. Since the closed form expression of probability density function does not exist, error probability can be obtained numerically. The variation of error probability is expressed in terms of skewness parameter and generalized signal to noise ratio (GSNR) due to the infinite variance property of α −stable noise. Furthermore, the receiver operating characteristics (ROC) are investigated in order to expose the variation of detection probability with respect to GSNR and noise parameters, especially skewness parameter.

The paper is organized as follows. The antipodal signal detection problem is described after α −stable distribution is defined in the next section. An approximate analytic expression is given in terms of probability of error with respect to variation of skewness. At the last section, receiver operating characteristics and dependence on noise parameters are discussed.

## 2. Material and Method

### 2.1. $\alpha$ −Stable distributions

One dimensional α −stable distribution is described by its characteristic function as given below [14]

$$\varphi(\omega) = \begin{cases} \exp\left\{-\sigma^{\alpha}|\omega|^{\alpha}\left(1 - j\beta\operatorname{sgn}(\omega)\tan\left(\frac{\pi\alpha}{2}\right) + j\mu\omega\right)\right\} & \text{if } \alpha \neq 1 \\ \exp\left\{-\sigma|\omega|\left(1 + j\beta\frac{2}{\pi}\operatorname{sgn}(\omega)\ln|\omega|\right) + j\mu\omega\right\} & \text{if } \alpha = 1 \end{cases} \quad (1)$$

where

$$\mathrm{sgn}(\omega) = \begin{cases} 1 & \text{if } \omega > 0 \\ 0 & \text{if } \omega = 0 \\ -1 & \text{if } \omega < 0 \end{cases}$$

The noise parameters characteristic exponent $\alpha$, skewness $\beta$, scale $\sigma$ and the shift parameter $\mu$, tune the impulsiveness, amount of asymmetry, intensity and the location, respectively. An alternative expression for intensity of the noise is dispersion $\gamma$, $\gamma = \sigma^{\alpha}$ [4]. If $\beta = 0$ and $\mu = 0$ the distribution is said to be symmetric. Since the shift on the location is not under consideration in this study, it is assumed $\mu = 0$.

The probability density function can be obtained by the relation [15]

$$f(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(\omega)\mathrm{e}^{-j\omega z}d\omega \quad (2)$$

The closed form expression of the stable distributions does not exist except for the cases Gaussian ($\alpha = 2$), Cauchy ($\alpha = 1$), and Levy distributions ($\alpha = 1/2, \beta = 1$) [15]. One of the distinctive properties of $\alpha$ −stable distributions is that fractional moments lower than characteristic exponent $\alpha$ are finite. It can be expressed in terms of a random variable $Z$ having $\alpha$ −stable distribution as [14]

$$\begin{aligned} \mathrm{E}|Z|^{\mathrm{p}} &< \infty \quad p < \alpha \\ \mathrm{E}|Z|^{\mathrm{p}} &= \infty \quad p \geq \alpha \end{aligned} \quad (3)$$

where E is the expectation operator. According to this property, variance is finite only for Gaussian noise case. This results in modifying the signal to noise ratio in terms of scale parameter, which is given in the next subsection. In the sequel, the antipodal signal detection problem is formulated by density functions under binary hypothesis testing.

### 2.2. Signal detection problem

In digital communication, antipodal signalling corresponds to baseband binary phase shift keying (BPSK) modulation. Considering the received signal carrying digital information in discrete time, the binary signal detection problem is given as

$$\begin{aligned} \mathcal{H}_1&: x = A + w \\ \mathcal{H}_0&: x = -A + w \end{aligned} \quad (4)$$

where the channel noise is generated from the density $w \sim f(z)$. An illustration of the PDF for both hypotheses is given in Figure 1. Differing from conventional Gaussian illustration in the literature, asymmetric behaviour is more apparent.

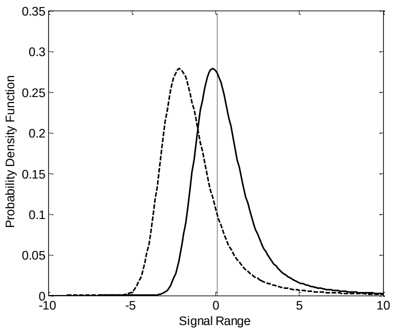


**Figure. 1**. Binary hypothesis testing under skewed $\alpha$ −stable noise with $A = 1$ and $\alpha = 1.5, \beta = 1, \sigma = 1$, solid - $\mathcal{H}_1$, dashed - $\mathcal{H}_0$ hypotheses

The error probability is given by (5)

$$\mathrm{P_e} = \mathrm{P}(\mathcal{H}_0|\mathcal{H}_1)\mathrm{P}(\mathcal{H}_1) + \mathrm{P}(\mathcal{H}_1|\mathcal{H}_0)\mathrm{P}(\mathcal{H}_0). \quad (5)$$

Since the message bits are assumed to be equally likely, $\mathrm{P}(\mathcal{H}_0) = \mathrm{P}(\mathcal{H}_1) = 1/2$. However, the conditional probabilities are not identical due to asymmetric behaviour of the noise. The general closed form expression of the density function cannot be obtained by characteristic function for an arbitrary characteristic exponent $\alpha$. The probability of error can be approximated by substituting (2) and (4) in (5) as follows

$$\mathrm{P_e} = \frac{1}{4\pi}\left[\int_{-\infty}^{0}\int_{-\infty}^{\infty} \varphi(\omega)\mathrm{e}^{-j\omega(x-A)}d\omega dx + \int_{0}^{\infty}\int_{-\infty}^{\infty} \varphi(\omega)\mathrm{e}^{-j\omega(x+A)}d\omega dx\right]. \quad (6)$$

Although the error probability is a function of noise intensity, it is more convenient to express variation of error with respect to signal to noise ratio. Since the $\alpha$ −stable noise has infinite variance for $\alpha < 2$, the term generalized signal to noise ratio (GSNR) [10] is defined by (7)

$$\mathrm{GSNR} = 10\log\frac{A^2}{\sigma^{\alpha}} \quad (7)$$

For sake of simplicity, the noise intensity is set $\sigma = 1$ and so that GSNR is reduced to tune the signal amplitude $A$ as $A = \sqrt{10^{\frac{\mathrm{GSNR}}{10}}}$. Then the error probability can be redefined as

$$P_e(\mathrm{GSNR}) = \frac{1}{4\pi}\left[\int_{-\infty}^{0}\int_{-\infty}^{\infty} e^{-|\omega|^{\alpha}\left(1-j\beta \mathrm{sgn}(\omega)\tan\left(\frac{\pi\alpha}{2}\right)\right)} e^{-j\omega\left(x-\sqrt{10^{\frac{\mathrm{GSNR}}{10}}}\right)} d\omega dx + \int_{0}^{\infty}\int_{-\infty}^{\infty} e^{-|\omega|^{\alpha}\left(1-j\beta \mathrm{sgn}(\omega)\tan\left(\frac{\pi\alpha}{2}\right)\right)} e^{-j\omega\left(x+\sqrt{10^{\frac{\mathrm{GSNR}}{10}}}\right)} d\omega dx\right] \quad (8)$$

where the impulsiveness and the skewness are assumed to lie within the intervals $0 < \alpha \leq 2$ and $-1 \leq \beta \leq 1$, respectively. In the next section, numerical simulations are given with respect to GSNR which illustrates the effect of noise parameters to error probability.

### 2.3. Error simulation

The simulations are performed in order to exhibit the effect of the parameters of the stable noise. The numerical integration in MATLAB environment is evaluated using Riemann sum method having step size 0.01 which corresponds to area under PDF 0.9964 for $\alpha = 0.8$ and 0.9989 for $\alpha = 1.5$. In Figure 2, the probability of error for antipodal signalling under symmetric $\alpha$ −stable (S$\alpha$S) distribution is illustrated with respect to various characteristic exponents. Note that the error curve for $\alpha = 2$ corresponds to Gaussian noise case. It is significant to observe that the error probability becomes apparently poorer when the noise characteristic is even slightly impulsive. Moreover, the error performance becomes dramatically worse especially for decreasing characteristic exponent $\alpha$, i.e., increasing impulsiveness.

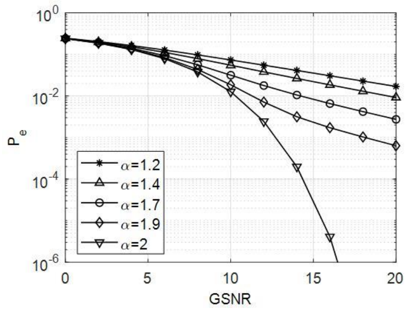


**Figure 2**. Probability of error under symmetric $\alpha$ −stable noise.

The effect of skewness on probability of error are investigated by tuning skewness for fixed characteristic exponent and tuning characteristic exponent for fixed skewness shown in Figure3a and Figure3b, respectively. It is quite apparent that, probability of error increases when the stable noise becomes more skewed for fixed characteristic exponent and more impulsive for fixed skewness.

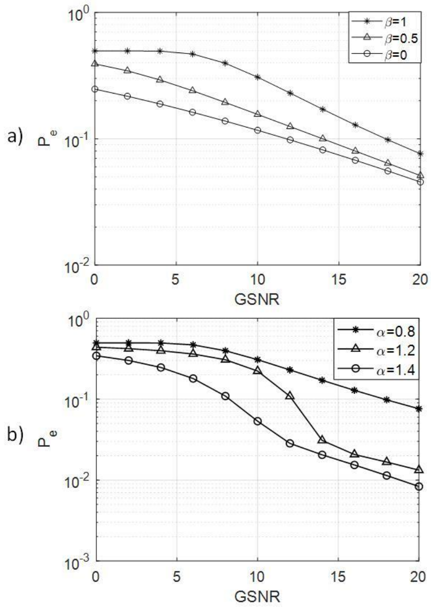


**Figure 3**. Effect of skewness and impulsiveness on error probability, a) Characteristic exponent $\alpha = 0.8$, b) Skewness parameter $\beta = 1$

On the other hand, the error probability is illustrated with respect to both skewness parameter $\beta$ and GSNR in Figure 4a-c. It is seen that the error probability obviously decreases with respect to GSNR. However, the error exhibits a valley with respect to $\beta$ and the lowest amount of error is obtained for $\beta = 0$, i.e., the noise is symmetric when GSNR is fixed. The valley becomes more apparent when the impulsiveness increases, i.e, the characteristic exponent decreases while the error performance in terms of GSNR become poorer at the same time.

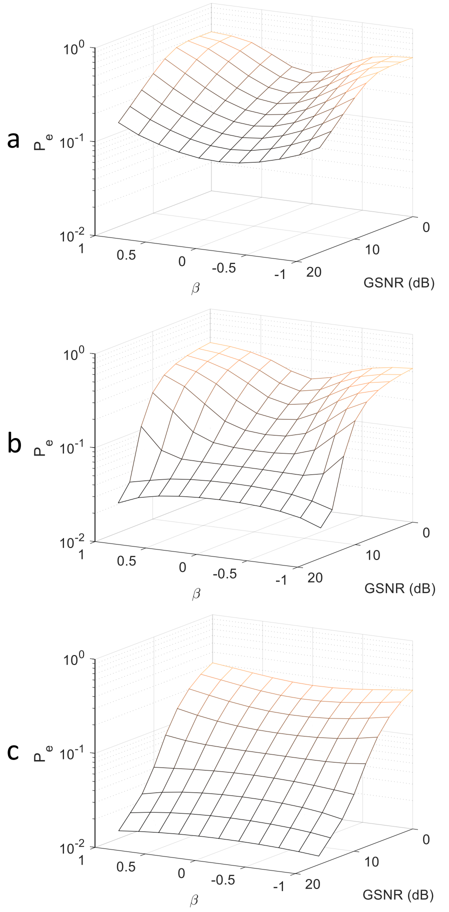


**Figure 4**. *Probability of error with respect to both skewness parameter β and GSNR.* *a)* $\alpha = 0.8$, *b)* $\alpha = 1.2$, *c)* $\alpha = 1.5$

In addition to the contribution on error probability, the receiver operating characteristics gives a clue about how the skewness has effect on detection probability versus false alarm probability, in the next section.

## 3. Receiver operating characteristics

The variation of probability of detection $P_D$ with respect to probability of false alarm $P_{FA}$ is essential to be analysed in order to exhibit the effect of both skewness and impulsiveness of the noise together. Noting from [16] that the test statistic T for antipodal signal detection problem can be tuned as $T \gtrless 0$ within the binary hypothesis testing problem. An approximate analytical expression for false alarm and detection probabilities are given in (9) and (10) as

$$P_{FA} = \Pr\{T > 0;\ \mathcal{H}_0\} = \frac{1}{2\pi}\left[\int_0^{\infty}\int_{-\infty}^{\infty} \varphi(\omega) e^{-j\omega(x+A)} d\omega dx\right]. \tag{9}$$

$$P_{D} = \Pr\{T > 0;\ \mathcal{H}_1\} = \frac{1}{2\pi}\left[\int_0^{\infty}\int_{-\infty}^{\infty} \varphi(\omega) e^{-j\omega(x-A)} d\omega dx\right]. \tag{10}$$

Before analysing the effect of skewness, the channel noise is fixed to be symmetric. Figure5a and Figure5b illustrate the ROC curve with respect to decreasing characteristic exponents, respectively. It can be said that the ROC performance is improved when GSNR increases, as expected. If the ROC performances are compared for fixed GSNR, it can be noticed that lower characteristic exponent results in poor ROC performance.

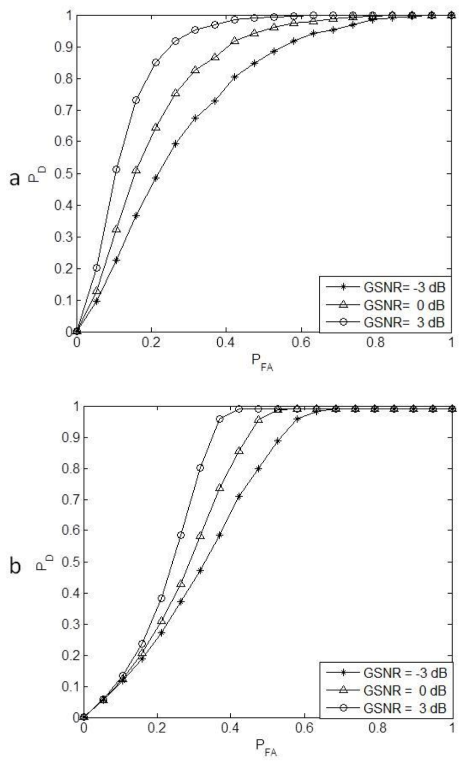


**Figure 5.** Variation of ROC curves with respect to different characteristic exponent of SαS noise. a) $\alpha = 1.5$, b) $\alpha = 0.8$

The effect of skewness on ROC curves is shown in Figure6a and Figure6b for different GSNR values.

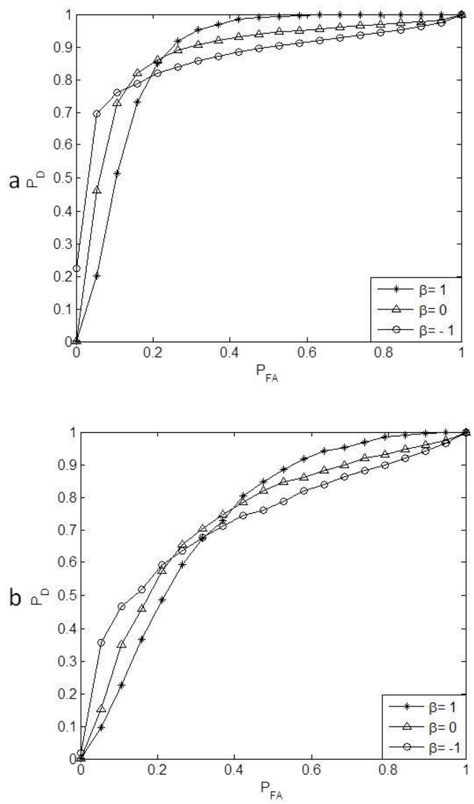


**Figure 6.** Variation of ROC curves with respect to different skewness for $\alpha = 1.5$, a) GSNR $= 3$dB, b) GSNR $= -3$dB

It appears that the higher probability of detection can be achieved than symmetrical case if the skewness of the channel noise is negative for small false alarm probability. However, this behaviour becomes reversed after a certain value of false alarm rate. The opposite comment is current for positive skewness. The detection probability is observed to be higher than SαS noise case for increasing false alarm rate while it is lower for small false alarm probabilities.

## 4. Discussion and Conclusion

This paper analyses the baseband digital communication problem using antipodal signals in skewed α −stable distributions. The findings can be treated with respect to the each noise parameter individually and generalized signal to noise ratio together. When the characteristic exponent is fixed, it is illustrated that the probability of error increases when the amount of positive or negative skewness of the α −stable noise increases. This behaviour becomes more apparent when the characteristic exponent decreases, corresponding to increasing impulsiveness. The error performance decreases when GSNR decreases, as expected.

The main contribution is concentrated on the effect of the skewness of the noise when the impulsiveness and GSNR are both fixed. It is exhibited that, the error probability has the lowest value when the noise has symmetrical behaviour corresponding to zero skewness ($\beta = 0$). According to this finding, it can be claimed that the error performance can be improved at the receiver if the channel noise is manipulated to exhibit resultant symmetric noise behaviour once the impulsiveness of the channel is known in advance. However it should be taken into account that this manipulation may result in increased total noise intensity at the receiver. Moreover, error performance may be observed to get worse if the characteristic exponent of the noise is decreased, which can be considered as more impulsive noise behaviour, consistent with earlier studies.

According to the ROC curves, higher detection probability can be achieved for a certain range of false alarm probability when the skewness is negative, and the opposite behaviour is observed for positive skewness. Finally, it is observed that detection performance can be tuned with respect to the selection of skewness and false alarm rate.

As a result, the present study generalizes the signal detection problem under non-Gaussian noise involving asymmetric distribution and is considered to give an insight for the investigation of antipodal signal detection problem under skewed α −stable noise for various modulation types in digital communication as the future work.

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