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Comparison of Rank-Based Tests for Ordered Alternative Hypotheses in Randomized Complete Block Designs

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Abstract

Nonparametric tests are useful when underlying distribution of a population is unknown or sample size is quiet small to satisfy assumptions of a traditional F test. Nonparametric tests have a good usage in a sample which consists of observations from various populations, as well. Randomized block designs are purposive when experimental subjects vary in natural heterogeneity. Nonparametric tests which are suitable for two-way ANOVA designs where the blocks containing observations which follow an increasing or a decreasing trend are main focus of this study. A recently proposed nonparametric test which was developed as an alternative to Jonckheere test is modified for ordered alternative hypotheses in randomized complete block designs. This modification test and several nonparametric tests for detecting ordered alternative hypotheses in randomized complete block designs are compared empirically in a broad set of Monte Carlo simulations under different conditions. A numerical example is provided to illustrate test procedures. The modified test provides better performance than Jonckheere test in terms of type I error and power values whereas Hollander test provides slightly better power values among the other test statistics. In terms of type I error values, it can be stated that the most conservative test is Jonckheere test whereas, estimated type 1 error values of the other test statistics are usually closer to nominal level of alpha.

1. INTRODUCTION

This article modifies a recently proposed rank-based test for randomized complete block designs for testing ordered alternative hypotheses. This recently proposed test was developed as a more powerful alternative to well-known Jonckheere test for ordered alternative hypotheses for independent groups [1]. The statistical analysis of randomized complete block designs (RCBD) can be carried out by parametric F tests if the underlying distributions are satisfied the normality assumption. Without knowledge of underlying distributions, testing treatment effects can be carried out by nonparametric tests. In this paper, we take into consideration nonparametric test statistics for ordered alternative hypotheses in RCBD. In testing the null hypothesis of no treatment effect in RB designs, an investigator may construct ordered alternative hypothesis with the prior knowledge of a monotonic relationship among treatment groups in order to increase power of a test statistic. A specific type of effect such as increasing or decreasing order occurs in a wide range of medical applications. For example, an investigator may wish to determine whether or not a continuous response increases with increased dosage in assessing efficacy of a new drug.

The model considered in this paper for a randomized block design is given in equation (1),

$$Y_{ij} = \mu + \tau_i + \beta_j + e_{ij} \qquad i = 1,\dots,n \qquad j = 1,\dots,b \tag{1}$$

where μ is common mean, τ_i are treatment effects, β_j are random block effects, n is the number of treatments of the j^{th} block, b is the number of blocks and e_{ij}'s are independent random

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variables with identical continuous distribution, F whose form may not be known. Null and alternative hypotheses are given as

$$\begin{aligned} &H_0 : \tau_1 = \tau_2 = \cdots = \tau_n = 0 \\ &H_1 : \tau_1 \le \tau_2 \le \cdots \le \tau_n \end{aligned} . \tag{2}$$

Jonckheere (1954) and Terpstra (1952) developed the nonparametric test for the nondecreasing ordered alternative based on the Mann Whitney testing procedure [2,3]. One of the most important test for ordered alternative hypothesis in randomized complete block design (RCBD) was developed by Page (1963) [4]. Hollander (1967) introduced a test statistic based on a sum of Wilcoxon signed-rank statistics for ordered alternatives [5]. A test statistic was suggested by Skillings and Wolfe (1978) by generalizing the Jonckheere statistic for ordered alternatives [6]. Alvo and Cabilio (1995) extended the Page and Jonckheere tests to the situation where the randomized blocks have missing observations [7]. Rayner and Best (1999) developed a test statistic by using Lancaster partition for ordered alternatives in an incomplete block design [8]. A new nonparametric test for detecting nondecreasing ordered alternatives is proposed by Terpstra and Magel (2003) [9]. Thas et al. (2012) used orthogonal trend contrasts for testing ranked data with ordered alternatives [10]. Zhang and Cabilio (2013) developed a generalized Jonckheere test against ordered alternatives for repeated measures in a randomized block design [11]. Akdur et al. (2016) generalized modified Jonckheere test against ordered alternatives for repeated measures in a randomized block design [12].

Also many applications of these rank-based tests can be found in clinical studies. Akilen et al. (2010) used Jonckheere’s test in order to demonstrate the effect of cinnamon on HbA1c [13]. Heffner et. al (1974) investigated the effect of the drug d-amphetamine sulfate on the behavior of rats in a RCBD [14]. Some practitioners may use Box-Cox procedure to find optimum transformation to satisfy normality instead of using nonparametric tests. After transforming the data, regression or ANOVA procedures can be used for testing null hypothesis against ordered alternative. For example, Nams et al. (1996) used isotonic ANOVA to assess an increasing relationship between production of pellets by snowshoe hares and amount of fertilizer on plots in a randomized block experiment by using log transformation of the data [15]. Shan et al. (2014) proposed a new idea for detecting a monotonic ordering by measuring rank difference between two observations from different independent groups [1]. This new nonparametric test not only captures the sign of the difference between observations as in commonly used Jonckheere test but it also makes use of the information in the value of rank difference. We extended this idea by modifying the new nonparametric test in the context of RCBD for detecting monotonic trend. The primary focus of the paper is to investigate performance of this modified rank-based test for RCBD. Our naive expectation is that including rank difference between observations will produce more powerful test than Jonckheere test in RCBD. Therefore, a simulation study is designed for comparing the modified test and the existing rank-based tests such as Page-type, Jonckheere and Hollander tests in terms of type I error and power values. The rest of this paper is organized as follows. In Section 2, Page-type, Jonckheere and Hollander tests are given briefly and Shan’s S test is modified and introduced for RCBD [1]. In simulation study section, performance of the modified test and other rank-based tests are compared emprically with regard to type I error and power values under different distribution and sample size conditions. A real example is included in application section to illustrate the rank-based nonparametric tests. Simulation findings are summarized with tables in results of simulation study section. Finally, following simulation results of the empirical levels and power analysis under different conditions, some discussions and comments are composed of the final section of the paper.

2. RANK-BASED TEST STATISTICS

Some nonparametric tests widely used for ordered alternative problem with randomized complete block design such as Hollander, Page’s L and Page’s T, generalized Jonckheere, generalized modified Jonckheere are given briefly in this section [5,10-12,16]. The recently proposed rank-based test statistic is modified for ordered alternative hypotheses in randomized complete block designs.

2.1. Hollander Test Statistic

Hollander (1967) test statistic based on Wilcoxon signed-rank test statistic is used to test null hypothesis against ordered alternative hypotheses in RCBD [5]. For each pair of (u,v) treatment, T_{uv} $\left(1 \le u < v \le n\right)$ signed-rank statistics are given as follows:

$T_{uv} = \sum_{j=1}^{b} R_{uv}^{j} \psi_{uv}^{j}$ where $X_{uv}^{j} = \left|Y_{ju} - Y_{jv}\right|$, $\psi_{uv}^{j} = \begin{cases} 1 & Y_{ju} < Y_{jv} \\ 1/2 & Y_{ju} = Y_{jv} \\ 0 & Y_{ju} > Y_{jv} \end{cases}$ and $R_{uv}^{(j)}$ is the rank of X_{uv}^{j} in the ranking from least to greatest of $\left[X_{uv}^{j}\right]_{j=1}^{b}$.

Hollander statistic based on T_{uv} is given,

$$H = \sum_{u=1}^{n-1} \sum_{v=u+1}^{n} T_{uv} \ . \tag{3}$$

Under the null distribution, $E(H) = \dfrac{bn(n-1)(b+1)}{8}$ and null variance of H is unknown and depends on the particular form of underlying continuous distribution of F .

2.2. Page's L and Page's T Test Statistics

Page test statistic is a well-known rank-based test for ordered alternative hypotheses in RCBD [4]. The original version of the test statistic is given as $P = \sum_{i=1}^{n} iR_i$ where $R_i = \sum_{j=1}^{b} R_{ij}$ and R_{ij} is the rank of response within block j at treatment i. Thas et al. (2012) provided a formula for the Page test statistic using orthogonal trend contrast for tied and untied data [10]. Page's L (PL) test statistic is given as

$$PL = \sqrt{c} \sum_{i=1}^{n} l_i \bar{R}_i \ / \ d \tag{4}$$

where $d^2 = \sum_{i=1}^{n} l_i^2$, $\bar{R}_i$ is the mean of the ranks for treatment i , l_i are the linear trend coefficients and $c = b(n-1)/(nV)$ for tied data $V = \sum_{ij} R_{ij}^2 / (bn) - (n+1)^2 / 4$ whereas for untied data, $V = (n^2 - 1)/12$ [17]. PL has an asymptotic N (0,1) distribution [16].

Best and Rayner (2015) proposed a new test statistic based on the PL, called as Page's T (PT), which derived from the orthogonal trend analysis used in ANOVA [16]. The alternative test statistic to PL is given below

$$PT = \sqrt{b} \sum_{i=1}^{n} \frac{l_i \bar{R}_i}{dS} \tag{5}$$

where S^2 is the error mean square from a randomized block ANOVA of the R_{ij}. If the data are normal, PT has an asymptotic student t distribution with $df = (b-1)(n-1)$. However, the data are ranks in this study. t_{df} student distribution approaches the standard normal distribution as the degree of freedom increase [16].

2.3. Jonckheere and Modified Jonckheere Test Statistics

The Jonckheere test statistic is also a well-known rank-based test for ordered alternative hypotheses [2]. This test statistic is based on Kendall's Tau correlation between a subject's responses and the alternative ordering where each subject is ranked within-block over treatment levels. $\mu_j(i)$ is the rank of the j^{th} block at treatment i, and $\mathrm{sgn}(\mu_j(m)-\mu_j(l))$ is either 1 or -1, depending on whether $\mu_j(m)>\mu_j(l)$ or $\mu_j(m)<\mu_j(l)$. The standardized version of Jonckheere test statistic for RCBD is given as

$$J=\frac{1}{b}\sum_{j=1}^{b}T_K(\mathrm{j}), \tag{6}$$

where $T_K(j)=\binom{n}{2}^{-1}A_K(j)$ and $A_K(j)=\sum_{l<m}^{n}\mathrm{sgn}(\mu_j(m)-\mu_j(l))$ [10]. Under null hypothesis, the asymptotic distribution of J is normal with zero mean and variance $bn(n-1)(2n+5)/18$ [11].
The standardized version of modified (weighted) Jonckheere test statistic is given as

$$MJ=\frac{1}{b}\sum_{j=1}^{b}T_K(j), \tag{7}$$

where $T_K(j)=\binom{n}{2}^{-1}A_K(j)$ and $A_K(j)=\sum_{l<m}^{n}(m-l)\mathrm{sgn}(\mu_j(m)-\mu_j(l))$ [12].

2.4. Modification of S Test Statistic

Shan et al. (2014) proposed a new nonparametric test which was called S test based on the rank difference between two observations in a given pair in order to improve test's efficiency by capturing actual rank differences from different independent groups for ordered alternative hypotheses [1]. For testing location parameters of k independent groups with the hypotheses $H_0=\mu_1=...=\mu_k$ against $H_1:\mu_1\le...\le\mu_k$, S test was provided as $S=\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}D_{ij}$ where $D_{ij}=\sum_{l=1}^{n_i}\sum_{m=1}^{n_j}Z_{ijlm}$, $Z_{ijlm}=\left(R_{jm}-R_{il}\right)\mathrm{I}\left(X_{jm}>X_{il}\right)$, R_{il} and R_{jm} denote the rank of the observation X_{il} and X_{jm} respectively [1].

In this study, S test statistic is modified for RCBD by using rank differences within each block to compose an overall test statistic. The modified S (MS) statistic is proposed as

$$MS=\frac{1}{b}\sum_{j=1}^{b}T_K(j) \tag{8}$$

where $\mu_j(i)$ is the rank of the j^{th} block at treatment i and indicator function $\mathrm{I}(\mu_j(m)>\mu_j(l))$ is either 1 or 0, depending on whether $\mu_j(m)>\mu_j(l)$ or $\mu_j(m)<\mu_j(l)$, $T_K(j)=\binom{n}{2}^{-1}S_K(j)$ and

$S_K(j)=\sum_{l<m}^{n}(\mu_j(m)-\mu_j(l))\mathrm{I}(\mu_j(m)>\mu_j(l))$.

3. SIMULATION STUDY

In the simulation study, Jonckheere-type tests, Page-type tests, Hollander (H) test and modification of S test for a randomized block layout are compared in terms of type I error and power of tests by conducting a broad Monte Carlo simulation study. Normal, log-normal, laplace and logistic distributions are taken into consideration. Also, four random block distributions are considered corresponding to the each error distribution: normal, log-normal, laplace and logistic distributions with zero mean and block variance $\sigma_\beta^2 = 1$.

In order to generate an increasing order of treatments, linear trend is assumed by using slope term θ below for comparing power performances of the tests

$$Y_{ij} = \theta i + \beta_j + e_{ij} \qquad i = 1,...,n \qquad j = 1,...,b. \tag{9}$$

For type I error comparions of the tests, θ slope value was taken as 0. For this linear trend model in equation (9) θ slope values were set as $\{0.2, 0.3, 0.4\}$ for power comparisons. By using the linear trend model in equation (9) with a certain θ slope value, a dataset which contains $n*b$ observations in increasing order were generated with random block effects from the corresponding error distribution with zero mean and $\sigma_\beta^2 = 1$. 10000 Monte Carlo samples were generated according to the model in equation (9) for each n , b, θ and for each distribution combination. The estimated power and type I error values of the test statistics were obtained based on the Monte Carlo simulated critical values of 10000 Monte Carlo samples.

Type I error and power values of modified Jonckheere are not reported in tables because its results are very similar to MS test statistic's type I error and power values. Table 1- Table 6 summarized Type I error and power values of the test statistics at $\alpha = 0.05$ level of significance. The simulation study was conducted in Cran R 3.4.3 and MSBVAR package was used to obtain datasets for the cases of normal distribution and log-normal distribution [17]. Interested readers may contact the corresponding author for requesting the R code.

Table 1. *Type I error values of tests under Normal and Lognormal distributions*

Distributions	*n*	*b*	J	MS	H	PL	PT
Under Normal	3	5	0.0314	0.0429	0.0534	0.0471	0.0494
		10	0.0329	0.0449	0.0510	0.0449	0.0488
		20	0.0417	0.0493	0.0506	0.0493	0.0513
	5	5	0.0524	0.0531	0.0508	0.0522	0.0527
		10	0.0379	0.0428	0.0506	0.0469	0.0476
		20	0.0449	0.0494	0.0496	0.0494	0.0507
	7	5	0.0399	0.0483	0.0485	0.0505	0.0547
		10	0.0505	0.0479	0.0484	0.0497	0.0499
		20	0.0423	0.0439	0.0477	0.0453	0.0453
Under Lognormal	3	5	0.0273	0.0390	0.0489	0.0428	0.0525
		10	0.0358	0.0424	0.0447	0.0378	0.0486
		20	0.0418	0.0501	0.0533	0.0501	0.0521
	5	5	0.0469	0.0486	0.0458	0.0500	0.0527
		10	0.0337	0.0436	0.0502	0.0486	0.0487
		20	0.0445	0.0458	0.0516	0.0485	0.0495
	7	5	0.0394	0.0484	0.0457	0.0501	0.0508
		10	0.0443	0.0496	0.0505	0.0506	0.0529
		20	0.0509	0.0510	0.0508	0.0524	0.0518

Table 2. *Type I error values of tests under Logistic and Laplace distributions*

Distributions	*n*	*b*	J	MS	H	PL	PT
Under Logistic	3	5	0.0263	0.0370	0.0333	0.0408	0.0470
		10	0.0340	0.0465	0.0438	0.0465	0.0472
		20	0.0413	0.0500	0.0496	0.0500	0.0505
	5	5	0.0516	0.0541	0.0510	0.0530	0.0541
		10	0.0563	0.0502	0.0528	0.0538	0.0540
		20	0.0462	0.0486	0.0495	0.0486	0.0501
	7	5	0.0382	0.0474	0.0448	0.0484	0.0467
		10	0.0483	0.0483	0.0509	0.0492	0.0501
		20	0.0506	0.0518	0.0546	0.0533	0.0519
Under Laplace	3	5	0.0283	0.0375	0.0506	0.0419	0.0453
		10	0.0281	0.0394	0.0389	0.0394	0.0398
		20	0.0411	0.0499	0.0478	0.0499	0.0525
	5	5	0.0315	0.0505	0.0449	0.0505	0.0493
		10	0.0551	0.0501	0.0557	0.0561	0.0560
		20	0.0447	0.0462	0.0555	0.0498	0.0501
	7	5	0.0397	0.0468	0.0476	0.0473	0.0483
		10	0.0432	0.0450	0.0487	0.0459	0.0471
		20	0.0484	0.0470	0.0472	0.0487	0.0481

Table 1 and 2 indicated that the estimated type I error values of each test were all within the interval $(0.026,\ 0.056)$, that was, within three standard deviation of the nominal level $\alpha = 0.05$. The estimated type I error values of Jonckheere test are slightly smaller than the nominal level $\alpha = 0.05$ for small sample sizes. It indicates that Jockheere test is a conservative for especially small sample sizes, i.e. the combination of $n = 3$ and $b = 5$. This situation for Jonckheere test seems to be corrected slightly when block sizes and treatment sizes increase in general. The similar results also are observed for MS and H tests for the smallest sample size cases of $n = 3$ and $b = 5$. As block and treatment sizes increase, the the estimated type I error values of five tests are closer to $\alpha = 0.05$ as seen in Table 1 and 2.

Table 3. *Power values of test statistics under Normal distribution with* $\theta = 0.2,\ 0.3,\ 0.4$

	n	b	J	MS	H	PL	PT
$\theta = 0.2$	3	5	0.0864	0.1094	0.1100	0.1188	0.1197
		10	0.1340	0.1670	0.2035	0.1670	0.1798
		20	0.2383	0.2762	0.3404	0.2762	0.2829
	5	5	0.2442	0.2952	0.3337	0.3099	0.3175
		10	0.5435	0.5379	0.6052	0.5515	0.5511
		20	0.7654	0.7903	0.8526	0.7906	0.7910
	7	5	0.6256	0.6744	0.7132	0.6752	0.6717
		10	0.8909	0.9160	0.9454	0.9172	0.9171
		20	0.9952	0.9967	0.9990	0.9967	0.9967
$\theta = 0.3$	3	5	0.1313	0.1664	0.2190	0.1807	0.1868
		10	0.2311	0.2832	0.3423	0.2832	0.2952
		20	0.4316	0.4791	0.5797	0.4791	0.4861
	5	5	0.5640	0.5804	0.6221	0.5763	0.5765
		10	0.7907	0.8345	0.8866	0.8349	0.8328
		20	0.9737	0.9806	0.9928	0.9806	0.9804
	7	5	0.9142	0.9327	0.9555	0.9349	0.9360
		10	0.9974	0.9977	0.9992	0.9977	0.9972
		20	1.0000	1.0000	1.0000	1.0000	1.0000
$\theta = 0.4$	3	5	0.1871	0.2323	0.3098	0.2450	0.2642
		10	0.3545	0.4188	0.5120	0.4188	0.4336
		20	0.6286	0.6768	0.7784	0.6768	0.6828
	5	5	0.7835	0.7967	0.8446	0.7942	0.7883
		10	0.9443	0.9653	0.9845	0.9664	0.9677
		20	0.9989	0.9991	0.9998	0.9991	0.9991
	7	5	0.9885	0.9936	0.9966	0.9936	0.9930
		10	1.0000	1.0000	1.0000	1.0000	1.0000
		20	1.0000	1.0000	1.0000	1.0000	1.0000

For normal distribution and $\theta = 0.2$, Hollander, PL and PT tests produce better power values than Jonckheere and MS tests. For $\theta = 0.3$ and $\theta = 0.4$ in normal distribution, Hollander test provides better power values among five tests. As block and treatment sizes increase, PL, PT, MS tests provide closer power values to each other as seen in Table 3.

Table 4. *Power values of test statistics under log-Normal distribution with* $\theta = 0.2,\ 0.3,\ 0.4$

	n	*b*	J	MS	H	PL	PT
$\theta = 0.2$	3	5	0.0769	0.1026	0.1000	0.1120	0.1174
		10	0.1361	0.1531	0.1828	0.1436	0.1714
		20	0.2360	0.2741	0.3166	0.2741	0.2805
	5	5	0.2517	0.2992	0.3152	0.2998	0.3116
		10	0.5422	0.5612	0.5902	0.5619	0.5676
		20	0.7709	0.7948	0.8333	0.7979	0.8018
	7	5	0.6824	0.6949	0.7016	0.6970	0.6895
		10	0.9054	0.9173	0.9246	0.9206	0.9180
		20	0.9951	0.9963	0.9973	0.9963	0.9965
$\theta = 0.3$	3	5	0.1283	0.1652	0.2008	0.1772	0.1944
		10	0.2361	0.2608	0.3101	0.2488	0.2911
		20	0.4230	0.4720	0.5328	0.4720	0.4788
	5	5	0.5612	0.5714	0.5939	0.5772	0.5801
		10	0.7839	0.8290	0.8663	0.8320	0.8304
		20	0.9751	0.9798	0.9876	0.9807	0.9809
	7	5	0.9039	0.9308	0.9343	0.9328	0.9317
		10	0.9976	0.9984	0.9990	0.9984	0.9985
		20	1.0000	1.0000	1.0000	1.0000	1.0000
$\theta = 0.4$	3	5	0.1873	0.2298	0.2377	0.2441	0.2626
		10	0.3524	0.4199	0.4537	0.4199	0.4267
		20	0.6274	0.6729	0.7406	0.6856	0.6927
	5	5	0.6889	0.7819	0.7992	0.7788	0.7863
		10	0.9441	0.9613	0.9724	0.9619	0.9639
		20	0.9994	0.9996	0.9997	0.9996	0.9996
	7	5	0.9936	0.9943	0.9954	0.9945	0.9938
		10	1.0000	1.0000	1.0000	1.0000	1.0000
		20	1.0000	1.0000	1.0000	1.0000	1.0000

As seen in Table 4 for the smallest sample size cases of $n = 3$ and $b = 5$ of $\theta = 0.2$, $\theta = 0.4$ in log-normal distribution, PT test provides slightly better power values than the other test statistics. In general, Hollander test produces slightly better power values for $\theta = 0.2$, $\theta = 0.3$ and $\theta = 0.4$ in log-normal distribution.

Table 5. *Power values of test statistics under Logistic distribution with* $\theta = 0.2,\ 0.3,\ 0.4$

	n	b	J	MS	H	PL	PT
$\theta = 0.2$	3	5	0.1046	0.1296	0.1590	0.1385	0.1552
		10	0.1816	0.2270	0.2400	0.2270	0.2331
		20	0.3257	0.3672	0.3940	0.3672	0.3725
	5	5	0.3456	0.4320	0.4012	0.4277	0.4199
		10	0.6119	0.6630	0.6724	0.6770	0.6823
		20	0.8944	0.9044	0.9171	0.9058	0.9073
	7	5	0.7614	0.7950	0.8040	0.7955	0.7886
		10	0.9644	0.9692	0.9679	0.9696	0.9702
		20	0.9998	0.9997	0.9996	0.9997	0.9998
$\theta = 0.3$	3	5	0.1655	0.2064	0.1990	0.2197	0.2259
		10	0.3817	0.3756	0.3999	0.3756	0.3892
		20	0.5636	0.5414	0.6349	0.5793	0.6019
	5	5	0.6942	0.6945	0.7014	0.6945	0.6941
		10	0.8908	0.9101	0.9241	0.9128	0.9162
		20	0.9953	0.9958	0.9964	0.9962	0.9963
	7	5	0.9562	0.9669	0.9679	0.9681	0.9672
		10	0.9996	0.9999	0.9996	0.9999	0.9999
		20	1.0000	1.0000	1.0000	1.0000	1.0000
$\theta = 0.4$	3	5	0.2453	0.2938	0.3458	0.3095	0.3330
		10	0.4665	0.5314	0.5792	0.5314	0.5456
		20	0.7683	0.8011	0.8427	0.8011	0.8037
	5	5	0.8603	0.8615	0.8695	0.8595	0.8526
		10	0.9887	0.9880	0.9911	0.9888	0.9890
		20	0.9999	0.9999	1.0000	0.9999	0.9999
	7	5	0.9965	0.9974	0.9976	0.9974	0.9975
		10	1.0000	1.0000	1.0000	1.0000	1.0000
		20	1.0000	1.0000	1.0000	1.0000	1.0000

For logistic distribution in Table 5, Hollander test produces better power values among the other test statistics. MS, PT, PL get closer to each other in terms of power values as block and treatment sizes increase. It is also seen that MS test has better performance than Jonckheere test.

Table 6. *Power of test statistics under Laplace distribution with* $\theta = 0.2,\ 0.3,\ 0.4$

	n	*b*	J	MS	H	PL	PT
$\theta = 0.2$	3	5	0.0591	0.0751	0.0976	0.0834	0.0929
		10	0.0819	0.1080	0.1205	0.1080	0.1212
		20	0.1231	0.1474	0.1668	0.1474	0.1543
	5	5	0.1274	0.1558	0.1661	0.1759	0.1771
		10	0.2232	0.2630	0.2964	0.2683	0.2773
		20	0.3981	0.4355	0.4790	0.4370	0.4418
	7	5	0.3007	0.3435	0.3561	0.3470	0.3450
		10	0.5405	0.5598	0.5865	0.5627	0.5634
		20	0.8044	0.8140	0.8385	0.8151	0.8168
$\theta = 0.3$	3	5	0.0694	0.0919	0.1182	0.0996	0.1110
		10	0.1166	0.1498	0.1738	0.1498	0.1512
		20	0.1977	0.2297	0.2665	0.2297	0.2320
	5	5	0.2867	0.2890	0.2760	0.2852	0.2924
		10	0.3966	0.4234	0.4746	0.4370	0.4486
		20	0.6838	0.7192	0.7621	0.7256	0.7246
	7	5	0.5268	0.5719	0.5983	0.5878	0.5809
		10	0.8306	0.8496	0.8726	0.8510	0.8524
		20	0.9777	0.9822	0.9882	0.9823	0.9822
$\theta = 0.4$	3	5	0.0939	0.1205	0.1555	0.1301	0.1445
		10	0.1665	0.2084	0.2293	0.2084	0.2104
		20	0.2999	0.3384	0.4014	0.3384	0.3462
	5	5	0.3206	0.3776	0.4427	0.4102	0.4057
		10	0.5986	0.6531	0.6985	0.6579	0.6587
		20	0.8807	0.8931	0.9318	0.8970	0.8994
	7	5	0.7470	0.7890	0.8110	0.7931	0.7900
		10	0.9643	0.9700	0.9794	0.9707	0.9693
		20	0.9995	0.9995	0.9999	0.9995	0.9995

As summarized in Table 6, Hollander test has better power values among the other test statistics. MS, PT, PL get closer to each other in terms of power values as block and treatment sizes increase.

4. REAL APPLICATION

A real dataset which used in this section was a result of experiment from a clinical research [18]. A subset of this dataset was used to illustrate Hollander test in the book by Hollander et al. (2013) [19]. The randomized block experiments were designed to investigate effect of load on forearm tremor frequency. Experimental subjects consisted of six males, age 21-43 years, free from neurological illness and the loads applied to the wrists have different level of weights. The model for mean of forearm tremor frequency of human subjects considered is given as $Y_{ij} = \mu + \tau_i + \beta_j + e_{ij}$, $i = 1,...,5$, $j = 1,...,6$. The null and alternative hypotheses for this example are given as

$$H_0 : \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0$$
$$H_1 : \tau_1 \le \tau_2 \le \tau_3 \le \tau_4 \le \tau_5$$

Table 7. *Mean forearm tremor frequency as a function of weight applied at the wrist*

	Treatment				
Subjects	1 (7.5 lb)	2 (5 lb)	3 (2.5 lb)	4 (1.25 lb)	5 (0 lb)
1	2.58	2.63	2.62	2.85	3.01
2	2.70	2.83	3.15	3.43	3.47
3	2.78	2.71	3.02	3.14	3.35
4	2.36	2.49	2.58	2.86	3.10
5	2.67	2.96	3.08	3.32	3.41
6	2.43	2.50	2.85	3.06	3.07

The observed values of J, MS, H, PL, PT statistics are 0.9333, 1.9666, 206.5, 4.7356, 17.4876 respectively. The p-values of J, MS, H, PL, PT statistics based on 100000 nonparametric bootstrap samples are 0.00001, 0.00001, 0.00002, 0.00001, 0.00005 respectively. According to the p-values of all test statistics, it is indicated that all test statistics are significant at the $\alpha = 0.05$ significance level. Therefore, it can be stated that adding load on a forearm decreases the tremor frequency of a forearm at the $\alpha = 0.05$ siginificance level according to bootstrap p values of test statistics.

5. DISCUSSION

In clinical dose-response trials, the ordered alternatives are the common alternative pattern. Jonckheere and Page type tests are the most common rank-based test statistics in many medical applications [2, 4]. Alternatively, Shan et al.'S statistic is modified for RCBD and compared the existing rank-based test statistics for randomized block design of ordered alternatives when the block effects have certain distributions which are determined based on the corresponding error distributions in this study [1]. This study not only aims to modify a rank-based test for ordered alternative hypotheses in RCBD but it also aims to gather the well-known rank-based nonparametric tests and compare their performances by using results of the Monte Carlo simulation. The estimated type I error values of the test statistics are generally closer to actual nominal level of alpha.

Comparing Jonckheere, modification of S, Hollander, Page's L and Page's T tests, there is little difference in the type I error values of the tests based on our simulation study. As the treatment and block sizes increase power values of the all test statistics increase. As the θ slope value increases, power values of all test statistics increase, as well. Under laplace distribution, the lowest power values of all test statistics generally are observed. Under normal distribution, the highest power values of all test statistics generally are observed. According to our simulation study, Hollander test seems more preferable especially in small sample size cases among these rank-based test statistics for testing the ordered alternative hypotheses in randomized complete block designs. Based on our simulation study, it is concluded that the most conservative test is Jonckheere test whereas estimated type 1 error values of the other test statistics are usually closer to nominal level of alpha in terms of type I error values. As aimed and expected, modification of S test provides better performance than Jonckheere test in terms of type 1 error and power values.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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