Available online: July 31, 2019

Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat.
Volume 68, Number 2, Pages 2298–2312 (2019)
DOI: 10.31801/cfsuasmas.000000
ISSN 1303–5991 E-ISSN 2618-6470

http://communications.science.ankara.edu.tr/index.php?series=A1

[1,2]-COMPLEMENTARY CONNECTED DOMINATION NUMBER OF GRAPHS-III

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Abstract. A set $S \subseteq V(G)$ in a graph G is said to be [1,2]-complementary connected dominating set if for every vertex $v \in V-S$, $1 \leq |N(v) \cap S| \leq 2$ and $< V-S >$ is connected. The minimum cardinality of [1,2]-complementary connected dominating set is called $[1,2]-$complementary connected domination number and is denoted by $\gamma_{[1,2]cc}(G)$. In this paper, we investigate $3-$regular graphs on twelve vertices for which $\gamma_{[1,2]cc}(G) = \chi(G) = 3$.

1. Introduction

Let $G(V, E)$ be simple and connected graph. For graph theoretic terminology we refer to Chartrand and Lesniak [1] and Haynes et.al [2]. In [6], V.R.Kulli and B.Janakiraman introduced the concept of nonsplit domination number of graph and characterized its bounds. In [3], Mustapha Chellali et.al, first studied the concept of $[1, 2]-$sets. In [7], Xiaojing Yang and Baoyindureng Wu, extended to the study of this parameter. In [4, 5], G.Mahadevan et.al, introduced the concept of $[1, 2]-$complementary connected domination and investigate $3-$regular graphs of order $n \leq 10$, whose $[1, 2]-$complementary connected domination number equals chromatic number equals three. In this paper, we investigate $3-$regular graphs on twelve vertices for which $\gamma_{[1,2]cc}(G) = \chi(G) = 3$.

2. 3-Regular graphs on twelve vertices

Let G be a connected cubic graph on twelve vertices for which $\chi(G) = \gamma_{[1,2]cc}(G) = 3$. Let $S = \{x, y, z\}$ be a $[1, 2]cc$-set. Since G is cubic, clearly $< S > \neq K_3, K_2 \cup$

Received by the editors: February 05, 2018; Accepted: June 25, 2018.

2010 *Mathematics Subject Classification.* 05C69.

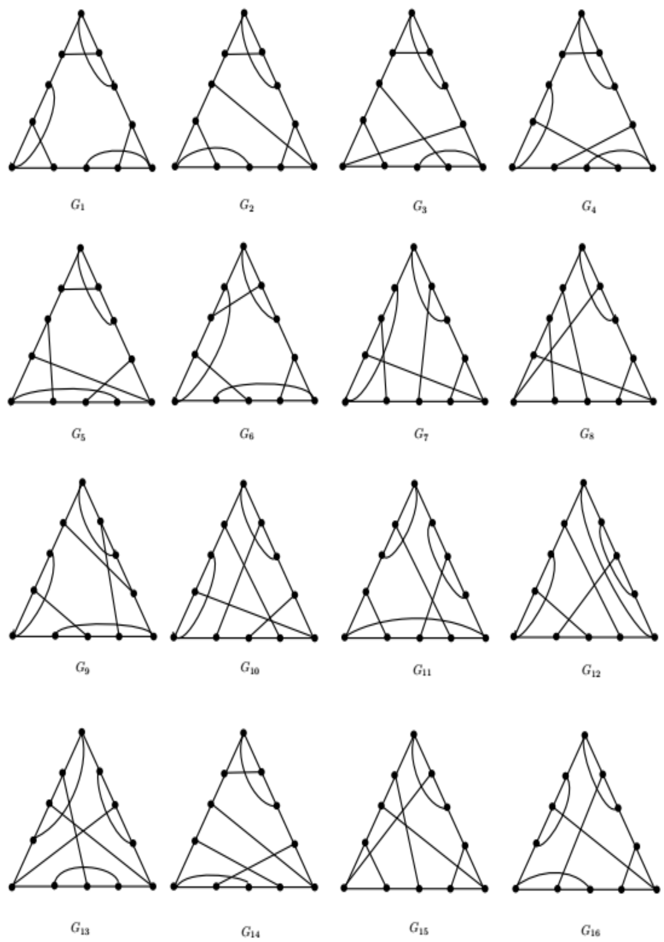
Key words and phrases. Complementary connected domination, [1,2]-sets, [1,2]-domination, [1,2]-complementary connected domination.

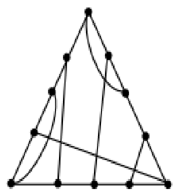
Submitted via International Conference on Current Scenario in Pure and Applied Mathematics [ICCSPAM 2018].

This research work was supported by Departmental Special Assistance, University Grants Commission, New Delhi and UGC-BSR Research fellowship in Mathematical Sciences- 2014-2015.

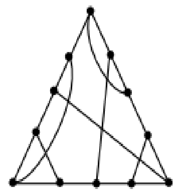
K_1, P_3. Hence $< S >= \bar{K}_3$. Let $S_1 = \{v_1, v_2, v_3\}$, $S_2 = \{v_4, v_5, v_6\}$ and $S_3 = \{v_7, v_8, v_9\}$. The following are only possible cases $< S_i >$, where $1 \leq i \leq 3$. Let $< S_1 >=< S_2 >=< S_3 >= P_3$; $< S_1 >=< S_2 >= P_3$, $< S_3 >= K_2 \cup K_1$; $< S_1 >=< S_2 >= P_3$, $< S_3 >= \bar{K}_3$; $< S_1 >= P_3$, $< S_2 >=< S_3 >= K_2 \cup K_1$; $< S_1 >= P_3$, $< S_2 >= K_2 \cup K_1$, $< S_3 >= \bar{K}_3$; $< S_1 >= P_3$, $< S_2 >=< S_3 >= \bar{K}_3$; $< S_1 >=< S_2 >=< S_3 >= K_2 \cup K_1$; $< S_1 >=< S_2 >= K_2 \cup K_1$, $< S_3 >= K_3$; $< S_1 >= K_2 \cup K_1$, $< S_2 >=< S_3 >= \bar{K}_3$; $< S_1 >=< S_2 >=< S_3 >= \bar{K}_3$.

Graphs G_i, where $1 \leq i \leq 32$





G_{17}



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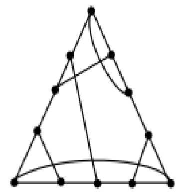

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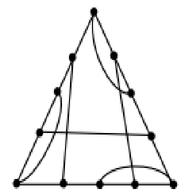
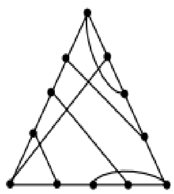
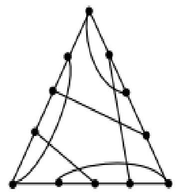


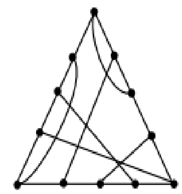
G_{20}



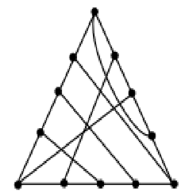
G_{21}



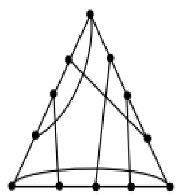
G_{22}



G_{23}



G_{24}



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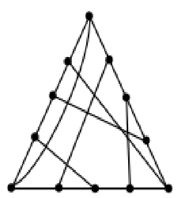
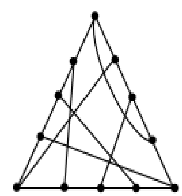
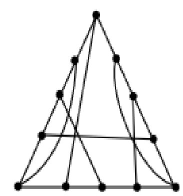


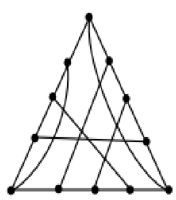
G_{26}



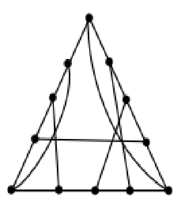
G_{27}



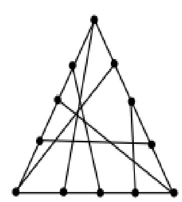
G_{28}



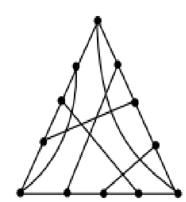
G_{29}



G_{30}



G_{31}



G_{32}

Prepositon 2.1. *If* $< S >= \bar{K}_3$ *and* $< S_1 >=< S_2 >=< S_3 >= P_3$*, then* $G \cong G_1$

Proof. Let $< S_1 >= P_3 = (v_1, v_2, v_3)$, $< S_2 >= P_3 = (v_4, v_5, v_6)$ and $< S_3 >= P_3 = (v_7, v_8, v_9)$. Let v_1 be adjacent to any one of $\{v_4, v_6, v_7, v_9\}$. Without loss of generality, let v_1 be adjacent to v_4. Now v_3 is adjacent to v_6 or v_7(or equivalently to v_9). If v_3 is adjacent to v_6, then no new graph exists. If v_3 is adjacent to v_7, then $G \cong G_1$. □

Prepositon 2.2. *If* $< S >= \bar{K}_3$ *and* $< S_1 >=< S_2 >= P_3$ *and* $< S_3 >= K_2 \cup K_1$, *then* $G \cong G_2$

Proof. Let $< S_1 >= P_3 = (v_1, v_2, v_3)$, $< S_2 >= P_3 = (v_4, v_5, v_6)$ and $< S_3 >= P_3 = (v_7, v_8, v_9)$, where $v_7v_8 \in E(S_3)$. Let v_1 be adjacent to any one of $\{v_4, v_6\}$ or any one of $\{v_7, v_8\}$ or v_9.

Case 1 $v_1v_4 \in E(G)$

Let v_3 be adjacent to v_6 or v_7(*or equivalently to* v_8) or v_9. If v_3 is adjacent to v_6, then no graph exists.

If v_3 is adjacent to v_7, then either v_6 is adjacent to v_8 or v_9. If v_6 is adjacent to v_8, then no new graph exists. If v_6 is adjacent to v_9, then no graph exists.

If v_3 is adjacent to v_9, then either v_6 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_6 is adjacent to v_7 or v_8 or v_9, then no graph exists.

Case 2 $v_1v_7 \in E(G)$

Let v_3 be adjacent to any one of $\{v_4, v_6\}$ or v_8 or v_9. Let v_3 be adjacent to v_4. Then v_6 is adjacent to any one of v_8 or v_9 and hence no graph exists.

Let v_3 be adjacent to v_8. Then v_9 is adjacent to v_4 and v_6. In this case, $< V - S >$ is disconnected and hence no graph exists.

Let v_3 be adjacent to v_9. Then v_4 is adjacent to v_8 or v_9. If v_4 is adjacent to v_8, then v_9 is adjacent to v_6 and hence $G \cong G_2$. If v_4 is adjacent to v_9, then v_6 is adjacent to v_8 and hence $G \cong G_2$.

Case 3 $v_1v_9 \in E(G)$

Let v_3 be adjacent to any one of $\{v_4, v_6\}$ or any one of $\{v_7, v_8\}$ or v_9. If v_3 is adjacent to v_4, then no new graph exists.

If v_3 is adjacent to v_7, then v_8 is adjacent to v_4(*or eqivalently to* v_6). If v_8 is adjacent to v_6, then v_9 is adjacent to v_4, so that $G \cong G_2$.

If v_3 is adjacent to v_9, then v_7 is adjacent to v_4 and v_8 is adjacent to v_6. In this case $< V - S >$ is disconnected and hence no graph exists. □

Prepositon 2.3. *If* $< S >= \bar{K}_3$ *and* $< S_1 >=< S_2 >= P_3$ *and* $< S_3 >= \bar{K}_3$, *then no graph exists.*

Proof. Let $< S_1 >= P_3 = (v_1, v_2, v_3)$, $< S_2 >= P_3 = (v_4, v_5, v_6)$ and $< S_3 >= \bar{K}_3 = (v_7, v_8, v_9)$. Let v_1 be adjacent to any one of $\{v_4, v_6\}$ or $\{v_7, v_8, v_9\}$

Case 1 Let v_1 be adjacent to v_4. Since G is cubic, v_3 cannot be adjacent to v_6 and hence v_3 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_3 is adjacent to v_7, then no new graph exists.

Case 2 Let v_1 be adjacent to v_7 and v_3 is adjacent to any one of $\{v_4, v_6\}$ or any one of $\{v_8, v_9\}$ or v_7. In all the above cases, no graph exists. □

Prepositon 2.4. *If* $< S >= \bar{K}_3$ *and* $< S_1 >= P_3$ *and* $< S_2 >=< S_3 >= K_2 \cup K_1$, *then* $G \cong G_i$, *where* $i = 3, 4$.

Proof. Let $< S_1 >= P_3 = (v_1, v_2, v_3)$, $< S_2 >= K_2 \cup K_1 = (v_4, v_5, v_6)$ and $< S_3 >= K_2 \cup K_1 = (v_7, v_8, v_9)$, where $v_4v_5, v_7v_8 \in E(G)$. Let v_1 be adjacent to $\{v_4, v_5, v_7, v_8\}$ or $\{v_6, v_9\}$.
Case 1 $v_1v_4 \in E(G)$.

In this case, v_3 must be adjacent to v_5 or v_6, $\{v_7, v_8\}$ or v_9.

Let v_3 be adjacent to v_5. Then v_6 is adjacent to either $\{v_7 \textit{ and } v_8\}$ or $\{v_7 \textit{ and } v_9\}$. If v_6 is adjacent to v_7 and v_8, then no graph exists. If v_6 is adjacent to v_7 and v_9, then no graph exists.

If v_3 is adjacent to v_6, then v_5 is adjacent to either v_9 or any one of $\{v_7, v_8\}$. If v_5 is adjacent to v_9, then no graph exists. If v_5 is adjacent to v_7, then no graph exists.
Case 2 $v_1v_6 \in E(G)$.

Let v_3 be adjacent to either v_9 or any one of $\{v_4, v_5\}$ or any one of $\{v_7, v_8\}$.

If v_3 is adjacent to v_9, then v_4 is adjacent to any one of $\{v_7, v_8\}$ or v_9. If v_4 is adjacent to v_9 and v_5 is adjacent to v_7, then v_6 is adjacent to v_8 and hence $G \cong G_3$. If v_4 is adjacent to v_7, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, and v_6 is adjacent to v_9, then $< V - S >$ is disconnected. If v_5 is adjacent to v_9, and v_6 is adjacent to v_8, then $G \cong G_3$. Let v_4 be adjacent to v_7. Then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, then v_6 is adjacent to v_9 and $< V - S >$ is disconnected. If v_5 is adjacent to v_9 and v_6 is adjacent to v_8, then $G \cong G_3$.

Let v_3 be adjacent to v_4. Then v_5 is adjacent to any one of $\{v_7, v_8\}$ or v_9. If v_5 is adjacent ot v_7, then v_6 is adjacent to v_8 or v_7. If v_6 is adjacent to v_8, then no graph exists. If v_5 is adjacent to v_9, then v_6 is adjacent to any one of $\{v_7, v_8\}$ or v_9. If v_6 is adjacent to v_7, then no graph exists. If v_6 is adjacent to v_9, then no graph exists.

Let v_3 be adjacent to v_7. Then v_4 is adjacent to v_8 or v_9. If v_4 is adjacent to v_8, then v_9 is adjacent to v_5 and v_6. Hence $G \cong G_4$. If v_4 is adjacent to v_9, then v_5 is adjacent to v_8 or v_9. Without loss of generality, let v_5 is adjacent to v_8, then v_6 is adjacent to v_9. Hence $G \cong G_4$. If v_5 is adjacent to v_9, then v_6 is adjacent to v_8. In this case $< V - S >$ is disconnected and hence no graph exists. □

Prepositon 2.5. *If* $< S >=< S_2 >= \bar{K}_3$ *and* $< S_1 >= P_3$ *and* $< S_3 >= K_2 \cup K_1$, *then* $G \cong G_{14}$.

Proof. Let $< S_1 >= P_3 = (v_1, v_2, v_3)$, $< S_2 >= \bar{K}_3 = (v_4, v_5, v_6)$ and $< S_3 >= K_2 \cup K_1 = (v_7, v_8, v_9)$, where $v_7v_8 \in E(G)$. Let v_1 be adjacent to any one of $\{v_4, v_5, v_6\}$ or $\{v_7, v_8\}$ or v_9.
Case 1 Let v_1 be adjacent to v_4. Then v_3 is adjacent to any one of $\{v_5, v_6\}$ or any one of $\{v_7, v_8\}$ or v_9 or v_4.

Let v_3 be adjacent to v_5. Then v_6 is adjacent to any one of $\{v_7, v_9\}$ or any one of $\{v_9, v_7\}$. If v_6 is adjacent to v_7 and v_8. Then v_9 is adjacent to v_4 and v_5. Hence $< V - S >$ is disconnected and hence no graph exists. If v_6 is adjacent to v_7 and v_9, then v_8 is adjacent to v_4 and v_9 is adjacent to v_5. Hence $G \cong G_{14}$.

Let v_3 be adjacent to v_7. Then v_8 is adjacent to any one of v_7 or $\{v_5, v_6\}$. If v_8 is adjacent to v_4, then v_9 must be adjacent to v_5 and v_6. Hence no graph exists. If v_8 is adjacent to v_5, then v_9 must be adjacent to $\{v_4, v_5\}$ or $\{v_4, v_6\}$. In both cases, no graph exists.

Let v_3 be adjacent to v_9. Then v_9 is adjacent to v_4 or any one of $\{v_5, v_6\}$. If v_9 is adjacent to v_4, then either v_5 or v_6 is adjacent to v_7 and v_8 or v_5 is adjacent to v_7 and v_6 is adjacent to v_8. In both cases no graph exists. If v_7 is adjacent to v_5 and v_8 is adjacent to $\{v_4 \textit{ and } v_5\}$ or $\{v_5 \textit{ and } v_6\}$. In both cases no graph exists.

Let v_3 be adjacent to v_4. Then v_9 is adjacent to v_5 and v_6, v_7 is adjacent to any one of $\{v_5, v_6\}$. In this case $< V - S >$ is disconnected and hence no graph exists.

Case 2 Let v_1 be adjacent to v_7. Then v_3 is adjacent to any one of $\{v_4, v_5, v_6\}$ or v_8 or v_9.

Let v_3 be adjacent to v_4. Then v_4 is adjacent to any one of v_8 or v_9. If v_4 is adjacent to v_8, then v_9 is adjacent to v_5 and v_6 and no graph exists. If v_4 is adjacent to v_9, then v_9 is adjacent to any one of $\{v_5, v_6\}$. If v_9 is adjacent to v_5 or v_6, then no new graph exists.

Let v_3 be adjacent to v_8. Then v_9 is adjacent to any two of $\{v_4, v_5, v_6\}$ and hence no new graph exists. Let v_3 be adjacent to v_9. Then no graph exists.

Case 3 Let v_1 be adjacent to v_9. Then v_3 is adjacent to any one of $\{v_4, v_5, v_6\}$ or any one of $\{v_7, v_8\}$ or v_9.

Let v_3 be adjacent to v_4. Then v_4 must be adjacent to any one of $\{v_7, v_8\}$ or v_9. In both cases, no graph exists.

Let v_3 be adjacent to v_7. Then v_4 is adjacent to v_8 and v_9. Hence no graph exists. Let v_3 be adjacent to v_9. In this case, $< V - S >$ is disconnected and hence no graph exists. □

Prepositon 2.6. *If* $< S >= \bar{K_3}$ *and* $< S_1 >= P_3$ *and* $< S_2 >=< S_3 >= \bar{K_3}$*, then* $G \cong G_5$.

Proof. Let $< S_1 >= P_3 = (v_1, v_2, v_3)$, $< S_2 >= \bar{K_3} = (v_4, v_5, v_6)$ and $< S_3 >= \bar{K_3} = (v_7, v_8, v_9)$. Let v_1 be adjacent to any one of $\{v_4, v_5, v_6, v_7, v_8, v_9\}$. Without loss of generality, let v_1 be adjacent to v_4. Then v_3 is adjacent to any one of $\{v_5, v_6\}$ or any one of $\{v_7, v_8, v_9\}$ or v_4.

Case 1 Let v_3 be adjacent to v_4. Then v_5 is adjacent to any two of $\{v_7, v_8, v_9\}$. In this case, $< V - S >$ is disconnected and hence no graph exists.

Case 2 Let v_3 be adjacent to v_5. Then v_4 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_4 is adjcent to v_7, then v_5 is adjacent to v_7 or $\{v_8, v_9\}$.

If v_5 is adjacent to v_7, then v_6 is adjacent to v_8 and v_9. Hence no graph exists. If v_5 is adjacent to v_8, then v_6 is adjacent to any one of $\{v_7, v_8\}$ or any one of $\{v_7, v_9\}$. In both cases, no graph exists.

Case 3 Let v_3 be adjacent to v_7. Then v_4 is adjacent to any one of $\{v_8, v_9\}$ or v_7.

Let v_4 be adjacent to v_8. Then v_5 is adjacent to any one of $\{v_7, v_8\}$ or any one of $\{v_8, v_9\}$. Let v_5 be adjacent to v_7 and v_8. Then v_6 is adjacent to v_9 and hence no graph exists. If v_5 is adjacent to v_7 and v_9, then v_6 is adjacent to v_8 and v_9. Hence $G \cong G_5$. □

Prepositon 2.7. *If* $< S >= \bar{K}_3$ *and* $< S_1 >=< S_2 >= K_2 \cup K_1$ *and* $< S_3 >= \bar{K}_3$, *then* $G \cong G_i$, *where* $6 \leq i \leq 13$.

Proof. Let $< S_1 >= K_2 \cup K_1 = (v_1, v_2, v_3)$, $< S_2 >= K_2 \cup K_1 = (v_4, v_5, v_6)$ and $< S_3 >= \bar{K}_3 = (v_7, v_8, v_9)$. Let v_1 be adjacent to any one of $\{v_4, v_5\}$ or v_6 or any one of $\{v_7, v_8, v_9\}$.

Case 1 Let v_1 be adjacent to v_4. Then v_2 is adjacent to v_5 or v_6 or any one of $\{v_7, v_8, v_9\}$. If v_2 is adjacent to v_5, then $< V - S >$ is disconnected and hence no graph exists in this case.

Subcase 1 Let $v_2v_6 \in E(G)$

Let v_3 be adjacent to $\{v_5, v_7\}$ or $\{v_6, v_7\}$ or $\{v_7, v_8, v_9\}$. If v_3 is adjacent to v_5 and v_6. Then $< V - S >$ is disconnected and hence no graph exists.

Let v_3 be adjacent to v_5 and v_7. Then v_6 is adjacent to any one of $\{v_7, v_8, v_9\}$ and hence no graph exists. If v_3 is adjacent to v_6 and v_4, v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$, then no graph exists in this case.

Let v_3 be adjacent to v_7 and v_8. Then v_5 is adjacent to any one of $\{v_7, v_8\}$ or v_9. If v_5 is adjacent to v_7, then v_6 is adjacent to v_8. In both cases no graph exists.

If v_5 is adjacent to v_9, then v_6 is adjacent to any one of $\{v_7, v_8, v_9\}$. Hence no graph exists for this case.

Subcase 2 Let $v_2v_7 \in E(G)$

Let v_3 be adjacent to both of $\{v_5, v_6\}$ or $\{v_5, v_7\}$ or $\{v_5, v_8\}$ or $\{v_6, v_7\}$ or $\{v_6, v_8\}$ or $\{v_7, v_8\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_5 and v_6, then v_6 is adjacent to any one of $\{v_7, v_8, v_9\}$. In this case, no graph exists. If v_3 is adjacent to v_5 and v_7, then v_6 is adjacent to any one of v_8 and v_9. Hence no graph exists.

If v_3 is adjacent to v_5 and v_8, then v_6 is adjacent to any two of $\{v_7, v_8, v_9\}$. Hence no graph exists. If v_3 is adjacent to v_6 and v_7, then v_5 is adjacent to any one of $\{v_8, v_9\}$. Hence no graph exists.

If v_3 is adjacent to v_6 and v_8, then v_5 is adjacent to any one of $\{v_8, v_7\}$ or v_9. In both cases no graph exists.

If v_3 is adjacent to v_7 and v_8, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, then v_6 is adjacent to v_9 and hence no graph exists. If v_5 is adjacent to v_9, then v_6 is adjacent to v_8 and v_9. Hence $G \cong G_6$.

If v_3 is adjacent to v_8 and v_9, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_5 is adjacent to v_7, then v_6 is adjacent to v_8 and v_9. Hence $G \cong G_7$.

Case 2 Let v_1 be adjacent to v_6. Then v_2 is adjacent to any one of $\{v_4, v_5\}$ or $\{v_7, v_8, v_9\}$ or v_6. If v_2 is adjacent to v_6, then $< V - S >$ is disconnected and hence no graph exists.

Subcase 1 Let $v_2v_4 \in E(G)$

Let v_3 be adjacent to v_5, v_6 or v_5, v_7 or v_6, v_7 or v_7, v_8 or v_7, v_9.

If v_3 is adjacent to v_5 and v_6, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_5 and v_7, then v_6 is adjacent to any one of $\{v_7, v_8, v_9\}$. Hence no graph exists.

If v_3 is adjacent to v_6 and v_7, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$. In this case, no graph exists. If v_3 is adjacent to v_7 and v_8, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$ and hence no graph exists.

Subcase 2 Let $v_2v_7 \in E(G)$

Let v_3 be adjacent to both of $\{v_5, v_4\}$ or $\{v_4, v_6\}$ or $\{v_4, v_7\}$ or $\{v_4, v_8\}$ or $\{v_7, v_6\}$ or $\{v_6, v_8\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_6, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$. In all the cases, no graph exists.

If v_3 is adjacent to v_4 and v_7, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, then v_6 is adjacent to v_9 or v_8 and hence no graph exists.

If v_3 is adjacent to v_4 and v_8, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$ and hence no graph exists for this case. If v_3 is adjacent to v_6 and v_7, then $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_6 and v_8, then v_4 is adjacent to v_7 or v_9. If v_4 is adjacent to v_7, then v_5 is adjacent to v_8 and hence no graph exists. If v_4 is adjacent to v_8, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$. In all the above cases, no graph exists.

Case 3 Let v_1 be adjacent to v_7. Then v_2 is adjacent to any one of $\{v_4, v_5\}$ or any one of $\{v_8, v_9\}$ or v_6 or v_7.

Subcase 1 Let $v_2v_4 \in E(G)$

Let v_3 be adjacent to both of $\{v_5, v_6\}$ or $\{v_5, v_7\}$ or $\{v_5, v_8\}$ or $\{v_6, v_7\}$ or $\{v_6, v_8\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_5 and v_6, then v_6 is adjacent to any one of $\{v_7, v_8, v_9\}$. Hence no graph exists. If v_3 is adjacent to v_5 and v_7, then in this case $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_5 and v_8, then v_6 is adjacent to any two of $\{v_7, v_8, v_9\}$. Hence no graph exists. If v_3 is adjacent to v_6 and v_7, then no graph exists. If v_3 is adjacent to v_6 and v_8, then no graph exists in this case.

If v_3 is adjacent to v_9 and v_8, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_5 is adjacent to v_7, then v_6 is adjacent to v_9 and v_8. Hence $< V - S >$ is disconnected and no graph exists. If v_5 is adjacent to v_8, then v_6 is adjacent to v_9 and v_7. Hence $G \cong G_8$.

Subcase 2 Let $v_2v_8 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_6\}$ or $\{v_4, v_7\}$ or $\{v_4, v_9\}$ or $\{v_6, v_7\}$ or $\{v_6, v_9\}$ or $\{v_7, v_8\}$ or $\{v_7, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then in this case $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_6, then no graph exists in this case.

If v_3 is adjacent to v_4 and v_5, then v_5 cannot be adjacent to v_8. Therefore v_5 is adjacent to v_9 and v_6 is adjacent to v_8, v_9. Hence $G \cong G_9$.

If v_3 is adjacent to v_4 and v_9, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_5 is adjacent to v_7, then v_6 is adjacent to v_8 and v_9. Hence $G \cong G_{10}$.

If v_3 is adjacent to v_6 and v_7, then v_4 is adjacent to v_8 or v_9. If v_4 is adjacent to v_8, then v_9 is adjacent to v_5 and v_6. Hence $G \cong G_9$. If v_4 is adjacent to v_9, then v_5 is adjacent to v_8 and v_6 is adjacent to v_9. Hence $G \cong G_9$.

If v_3 is adjacent to v_6 and v_9, then v_4 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_4 is adjacent to v_7, then v_5 is adjacent to v_8 and v_6 is adjacent to v_9. Hence $G \cong G_{11}$.

If v_3 is adjacent to v_7 and v_8, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_7 and v_9, then v_4 is adjacent to v_8 and v_5 is adjacent to v_9 and hence no graph exists in this case.

Subcase 3 Let $v_2v_6 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_6\}$ or $\{v_4, v_7\}$ or $\{v_4, v_8\}$ or $\{v_6, v_7\}$ or $\{v_6, v_8\}$ or $\{v_7, v_8\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_6, then v_5 is adjacent to any one of $\{v_8, v_9\}$ or v_7. In both cases, no graph exists.

If v_3 is adjacent to v_4 and v_7, then v_5 is adjacent to any one of $\{v_8, v_9\}$. If v_5 is adjacent to v_8, then no graph exists.

If v_3 is adjacent to v_4 and v_8, then v_5 is adjacent to any one of $\{v_8, v_7\}$ or v_9. In both cases no graph exists. If v_3 is adjacent to v_6 and v_7, then in this case $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_6 and v_8, then v_4 is adjacent to any one of $\{v_8, v_7\}$ or v_9. In both cases no graph exists.

If v_3 is adjacent to v_7 and v_8, then v_4 is adjacent to v_8 or v_9. If v_4 is adjacent to v_8, then v_9 is adjacent to v_5 and v_6. Hence $G \cong G_{12}$. If v_4 is adjacent to v_9, then v_5 is adjacent to v_8 and v_6 is adjacent to v_9. Hence $G \cong G_{12}$.

If v_3 is adjacent to v_9 and v_8, then v_4 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_4 is adjacent to v_7, then v_5 is adjacent to any one of $\{v_8, v_9\}$. Without loss of generality, let v_5 be adjacent to v_8 and v_6 be adjacent to v_9. Hence $G \cong G_{13}$.

Subcase 3 $v_2v_7 \in E(G)$

Let v_2 be adjacent to v_7. In this case, $< V - S >$ is disconnected and hence no graph exists. □

Prepositon 2.8. *If* $< S >= \bar{K}_3$ *and* $< S_1 >=< S_2 >=< S_3 >= K_2 \cup K_1$*, then* $G \cong G_i$*, where* $15 \leq i \leq 21$*.*

Proof. Let $< S_1 >= K_2 \cup K_1 = (v_1, v_2, v_3)$, $< S_2 >= K_2 \cup K_1 = (v_4, v_5, v_6)$ and $< S_3 >= K_2 \cup K_1 = (v_7, v_8, v_9)$ and $v_1v_2, v_4v_5, v_7v_8 \in E(G)$. Let v_1 be adjacent to any one of $\{v_4, v_5, v_7, v_8\}$ or $\{v_6, v_9\}$.

Case 1 Let v_1 be adjacent to v_4. Then v_2 is adjacent to any one of $\{v_7, v_8\}$ or v_5 or v_6 or v_9.

Subcase 1 Let $v_2v_7 \in E(G)$

Let v_3 be adjacent to both of $\{v_5, v_6\}$ or $\{v_5, v_8\}$ or $\{v_6, v_9\}$. If v_3 is adjacent to v_5 and v_6, then no graph exists. If v_3 is adjacent to v_5 and v_8, then no graph exists.

If v_3 is adjacent to v_6 and v_9, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, then v_6 is adjacent to v_9. In this case $< V - S >$ is disconnected hence no graph exists. If v_5 is adjacent to v_9, then v_6 is adjacent to v_8 and hence $G \cong G_{15}$.

Subcase 2 Let $v_2v_5 \in E(G)$

In this case, $< V - S >$ is disconnected hence no graph exists.

Subcase 3 Let $v_2v_6 \in E(G)$

Let v_3 be adjacent to both of $\{v_5, v_6\}$ or $\{v_5, v_7\}$ or $\{v_5, v_9\}$ or $\{v_6, v_7\}$ or $\{v_6, v_9\}$ or $\{v_7, v_8\}$ or $\{v_7, v_9\}$.

If v_3 is adjacent to v_5 and v_6, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_5 and v_7, then v_6 must be adjacent to any one of $\{v_7, v_8, v_9\}$. Hence no graph exists.

If v_3 is adjacent to v_5 and v_9, then v_6 must be adjacent to any one of $\{v_7, v_8, v_9\}$. Hence no graph exists. If v_3 is adjacent to v_6 and v_7, then v_5 must be adjacent to any one of v_8 or v_9. In both cases no graph exists.

If v_3 is adjacent to v_6 and v_9, then v_5 must be adjacent to any one of $\{v_7, v_8, v_9\}$. Hence no graph exists. If v_3 is adjacent to v_7 and v_8, then $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_7 and v_9, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8 and v_6 is adjacent to v_9, then $G \cong G_{17}$. If v_5 is adjacent to v_9 and v_6 is adjacent to v_8, then $G \cong G_{18}$.

Subcase 4 Let $v_2v_9 \in E(G)$

Let v_3 be adjacent to both of $\{v_5, v_6\}$ or $\{v_5, v_7\}$ or $\{v_5, v_9\}$ or $\{v_6, v_7\}$ or $\{v_6, v_9\}$ or $\{v_7, v_8\}$ or $\{v_7, v_9\}$.

If v_3 is adjacent to v_5 and v_6, then v_6 is adjacent to any one of $\{v_7, v_8, v_9\}$ and hence no graph exists. If v_3 is adjacent to v_5 and v_7, then v_6 is adjacent to v_8 and v_9 and hence $G \cong G_{18}$.

If v_3 is adjacent to v_5 and v_9, then $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_6 and v_7, then v_5 is adjacent to any one of v_8 or v_9. If v_5 is adjacent to any one of v_8, then v_6 is adjacent to v_9. Hence $G \cong G_{15}$. If v_5 is adjacent to any one of v_9, then v_6 is adjacent to v_8. In this case, $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_6 and v_9, then v_5 is adjacent to v_7 and v_6 is adjacent to v_8. Hence $G \cong G_{19}$. If v_3 is adjacent to v_7 and v_8, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_7 and v_9, then no graph exists.

Case 2 Let v_1 be adjacent to v_6. Then v_2 is adjacent to $\{v_4, v_5\}$ or v_6 or $\{v_7, v_8\}$ or v_9. If v_2 is adjacent to v_6, then $< V - S >$ is disconnected and hence no graph exists in this case.

Subcase 1 Let $v_2v_4 \in E(G)$

Let v_3 be adjacent to both of $\{v_5, v_6\}$ or $\{v_5, v_7\}$ or $\{v_5, v_9\}$ or $\{v_6, v_7\}$ or $\{v_6, v_9\}$ or $\{v_7, v_8\}$ or $\{v_7, v_9\}$.

If v_3 is adjacent to v_5 and v_6, then $< V - S >$ is disconnected and hence no graph exists in this case. If v_3 is adjacent to v_5 and v_7, then no graph exists. If v_3 is adjacent to v_5 and v_9, then no graph exists.

If v_3 is adjacent to v_6 and v_7, then no graph exists. If v_3 is adjacent to v_6 and v_9, then no graph exists. If v_3 is adjacent to v_8 and v_7, then $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_9 and v_7, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, then v_6 is adjacent to v_9. Hence $G \cong G_{19}$. If v_5 is adjacent to v_9, then v_6 is adjacent to v_8 and hence $G \cong G_{20}$.

Subcase 2 Let $v_2v_7 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_6\}$ or $\{v_4, v_8\}$ or $\{v_4, v_9\}$ or $\{v_6, v_8\}$ or $\{v_6, v_9\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_6, then v_5 is adjacent to v_8 or v_9. Hence no graph exists. If v_3 is adjacent to v_4 and v_8, then v_9 is adjacent to v_5 or v_6. Hence $G \cong G_{20}$.

If v_3 is adjacent to v_4 and v_9, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, v_6 is adjacent to v_9. Hence $G \cong G_{15}$. If v_5 is adjacent to v_9 and v_6 is adjacent to v_8, then in this case $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_6 and v_8 and v_9 is adjacent to v_4 or v_5, then $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_6 and v_9, then v_8 is adjacent to any one of $\{v_4, v_5\}$. If v_8 is adjacent to v_4 and v_9 is adjacent to v_5, then $G \cong G_{21}$. If v_3 is adjacent to v_8 and v_9, then no graph exists.

Subcase 3 Let $v_2v_9 \in E(G)$

Let v_3 be adjacent to any two of $\{v_4, v_5, v_7, v_8\}$ or both of $\{v_4, v_6\}$ or both of $\{v_6, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_6, then no graph exists. If v_3 is adjacent to v_6 and v_9, then $< V - S >$ is disconnected and hence no graph exists in this case. □

Prepositon 2.9. *If* $< S >= \bar{K_3}$ *and* $< S_1 >= K_2 \cup K_1$ *and* $< S_3 >=< S_2 >= \bar{K_3}$*, then* $G \cong G_i$*, where* $i = 22, 23$.

Proof. Let $< S_1 >= K_2 \cup K_1 = (v_1, v_2, v_3)$, $< S_2 >= K_2 \cup K_1 = (v_4, v_5, v_6)$ and $< S_3 >= \bar{K_3} = (v_7, v_8, v_9)$, where $v_1v_2 \in E(G)$. Let v_1 be adjacent to any one of $\{v_4, v_5, v_6, v_7, v_8, v_9\}$.

If v_1 is adjacent to v_4, then v_2 is adjacent to any one of $\{v_5, v_6\}$ or any one of $\{v_7, v_8, v_9\}$ or v_4. If v_2 is adjacent to v_4, then in this case $< V - S >$ is disconnected and hence no graph exists.

Case 1 If v_2 is adjacent to v_5, then v_3 is adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_6\}$ or $\{v_4, v_7\}$ or $\{v_6, v_7\}$ or $\{v_7, v_8\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_6, then no graph exists in this case.

If v_3 is adjacent to v_4 and v_7, then v_5 is adjacent to any one of $\{v_8, v_9\}$ or v_7. If v_5 is adjacent to v_7, then no graph exists. If v_5 is adjacent to v_8, then no graph exists.

If v_3 is adjacent to v_6 and v_7, then v_4 is adjacent to v_7 or any one of $\{v_8, v_9\}$. If v_4 is adjacent to v_7, then v_5 is adjacent to v_8 and v_6 is adjacent to v_9. Hence no graph exists. If v_4 is adjacent to v_8, then v_5 is adjacent to any one of $\{v_7, v_8\}$ or v_9. In both cases no graph exists.

If v_3 is adjacent to v_7 and v_8, then v_4 is adjacent to any one of v_9 or $\{v_7, v_8\}$. If v_4 is adjacent to v_7, then v_5 is adjacent to v_8 or v_9. If v_5 is adjacent to v_8, then no graph exists. If v_5 is adjacent to v_9, then v_6 is adjacent to v_8 or v_9. Hence $G \cong G_{22}$.

If v_4 is adjacent to v_9, then v_5 is adjacent to any one of $\{v_7, v_8, v_9\}$. If v_5 is adjacent to v_7, then v_6 is adjacent to v_8 and v_9, then $G \cong G_{23}$.

Case 2

If v_2 is adjacent to v_7, then v_3 is adjacent to both of $\{v_4, v_7\}$ or $\{v_4, v_5\}$ or $\{v_5, v_6\}$. If v_3 is adjacent to v_4 and v_7, then $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_4 and v_5, then v_5 is adjacent to v_7 or any one of $\{v_8, v_9\}$. If v_5 is adjacent to v_7, then $< V - S >$ is disconnected and hence no graph exists. If v_5 is adjacent to v_8, then no graph exists.

If v_3 is adjacent to v_5 and v_6, then v_4 is adjacent to v_7 or any one of $\{v_8, v_9\}$. If v_4 is adjacent to v_7, then $< V - S >$ is disconnected and hence no graph exists. If v_4 is adjacent to v_8, then no graph exists. □

Prepositon 2.10. *If $< S > = < S_1 > = < S_2 > = < S_3 > = \bar{K_3}$, then $G \cong G_i$, where $24 \leq i \leq 32$.*

Proof. Let $< S_1 > = \bar{K_3} = (v_1, v_2, v_3)$, $< S_2 > = \bar{K_3} = (v_4, v_5, v_6)$ and $< S_3 > = \bar{K_3} = (v_7, v_8, v_9)$. Let v_1 be adjacent to any one of $\{v_4, v_5\}$ or any one of $\{v_6, v_7\}$.

Case 1 Let v_1 be adjacent to v_4 and v_5. Then v_2 is adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_6\}$ or $\{v_4, v_7\}$ or $\{v_6, v_7\}$ or $\{v_7, v_8\}$.

Subcase 1 $v_2v_4, v_2v_5 \in E(G)$

Let v_2 be adjacent to v_4 and v_5. In this case, $< V - S >$ is disconnected and hence no graph exists.

Subcase 2 $v_2v_4, v_2v_6 \in E(G)$

If v_2 is adjacent to v_4 and v_6, then v_3 is adjacent to both of $\{v_5, v_6\}$ or $\{v_5, v_7\}$ or $\{v_7, v_8\}$. If v_3 is adjacent to v_5 and v_6, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_5 and v_7, then no graph exists. If v_3 is adjacent to v_8 and v_7, then no graph exists.

Subcase 3 $v_2v_4, v_2v_7 \in E(G)$

Let v_3 be adjacent to both of $\{v_6, v_5\}$ or $\{v_5, v_7\}$ or $\{v_5, v_8\}$ or $\{v_6, v_7\}$ or $\{v_6, v_8\}$ or $\{v_7, v_8\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_5 and v_6, then no graph exists. If v_3 is adjacent to v_5 and v_7, then $< V - S >$ is disconnected and hence no graph exists.

If v_3 is adjacent to v_5 and v_8, then no graph exists. If v_3 is adjacent to v_6 and v_7, then no graph exists. If v_3 is adjacent to v_6 and v_8, then no graph exists.

If v_3 is adjacent to v_7 and v_8. Since G is cubic, v_5 cannot be adjacent to v_8. Hence v_5 is adjacent to v_9 and v_6 is adjacent to v_9 and v_8. Hence $G \cong G_{24}$.

If v_3 is adjacent to v_9 and v_8, then v_5 is adjacent to v_7 and v_6 is adjacent to v_8 and v_9. In this case, $< V - S >$ is disconnected and hence no graph exists.

Subcase 4 $v_2v_6, v_2v_7 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_7\}$ or $\{v_7, v_8\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_7, then no graph exists.

If v_3 is adjacent to v_7 and v_8, then v_4 must be adjacent to v_8 or v_9. If v_4 is adjacent to v_8, then v_9 is adjacent to v_5 and v_6. Hence $G \cong G_{25}$. If v_4 is adjacent to v_9, then v_5 is adjacent to v_8 and v_6. Hence $G \cong G_{26}$.

If v_3 is adjacent to v_9 and v_8, then v_4 must be adjacent to v_7, v_5 is adjacent to v_8 and v_6 is adjacent to v_9. Hence $G \cong G_{27}$.

Subcase 5 $v_2v_7, v_2v_8 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_6\}$ or $\{v_4, v_7\}$ or $\{v_6, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_6, then no graph exists.

Let v_3 is adjacent to v_4 and v_7. Since G is cubic, v_5 cannot be adjacent to v_8. If v_5 is adjacent to v_9, then v_6 is adjacent to v_8 and v_9. Hence $G \cong G_{24}$.

If v_3 is adjacent to v_6 and v_9, then v_4, v_5, v_6 are adjacent to v_7, v_8, v_9. Hence $< V - S >$ is disconnected and hence no graph exists.

Case 2 If v_1 is adjacent to v_6 and v_7, then v_2 is adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_6\}$ or $\{v_6, v_7\}$ or $\{v_6, v_8\}$. If v_2 is adjacent to v_6 and v_7, then $< V - S >$ is disconnected and hence no graph exists.

Subcase 1 $v_2v_4, v_2v_5 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_7\}$ or $\{v_4, v_8\}$ or $\{v_7, v_8\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_7, then no graph exists.

If v_3 is adjacent to v_4 and v_8, then no graph exists. If v_3 is adjacent to v_7 and v_8, then v_4 is adjacent to v_8 and v_9 is adjacent to v_5 and v_6. Hence $G \cong G_{28}$.

If v_3 is adjacent to v_9 and v_8, then v_4 is adjacent to v_7 and v_5 is adjacent to v_8 and v_6. Hence $G \cong G_{29}$.

Subcase:2 $v_2v_4, v_2v_6 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_7\}$ or $\{v_4, v_8\}$ or $\{v_5, v_7\}$ or $\{v_8, v_5\}$ or $\{v_7, v_8\}$ or $\{v_8, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then no graph exists. If v_3 is adjacent to v_4 and v_7, then $< V - S >$ is disconnected and hence no graph exists. If v_3 is adjacent to v_4 and v_8, then no graph exists.

If v_3 is adjacent to v_7 and v_5, then no graph exists. If v_3 is adjacent to v_8 and v_5, then no graph exists. Let v_3 be adjacent to v_7 and v_8. Since G is cubic v_4 cannot be adjacent to v_8. Hence v_4 is adjacent to v_9 and v_5 is adjacent to v_8, v_9. Hence $G \cong G_{24}$.

If v_3 is adjacent to v_8 and v_9, then v_4 is adjacent to v_7, v_5 is adjacent to v_8 and v_9. Hence $G \cong G_{30}$.

Subcase:3 $v_2v_6, v_2v_8 \in E(G)$

Let v_3 be adjacent to both of $\{v_4, v_5\}$ or $\{v_4, v_7\}$ or $\{v_4, v_9\}$ or $\{v_7, v_8\}$ or $\{v_7, v_9\}$.

If v_3 is adjacent to v_4 and v_5, then no graph exists. Let v_3 be adjacent to v_4 and v_7. Since G is cubic, v_4 cannot be adjacent to v_8. Hence v_4 is adjacent to v_9 and v_5 is adjacent to v_8 and v_9. Hence $G \cong G_{31}$.

If v_3 is adjacent to v_4 and v_9, then v_4 is adjacent to v_8 and v_9. Hence $G \cong G_{32}$. If v_3 is adjacent to v_7 and v_8, then no graph exists. If v_3 is adjacent to v_7 and v_9, then no graph exists. □

Theorem 2.1. *Let G be a* $3-$*regular graph of order twelve. Then* $\chi(G) = \gamma_{[1,2]cc}(G) = 3$ *if and only if* $G \cong G_i$, *where* $1 \leq i \leq 32$.

Proof. If G is any one of the graphs G_i, where $1 \leq i \leq 32$ as in the figure 1, then clearly verified that $\chi(G) = \gamma_{[1,2]cc}(G) = 3$. Conversly, assume that $\chi(G) = \gamma_{[1,2]cc}(G) = 3$. Then the proof follows from proposition 2.1 to 2.10. □

Conclusion. In this paper we investigated $3-$regular graphs of order 12, whose $[1, 2]-$Complementary connected domination and chromatic number are eqaual to three.

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