Available online at www.alphanumericjournal.com

alphanumeric journal
The Journal of Operations Research, Statistics, Econometrics and Management Information Systems

Volume 8, Issue 2, 2020

Received: December 19, 2019
Accepted: September 24, 2020
Published Online: December 31, 2020

AJ ID: 2020.08.02.OR.04
DOI: 10.17093/alphanumeric.659121
Research Article

A Different Approach to Nurse Scheduling Problem: Lagrangian Relaxation

Yücel Öztürkoğlu, Ph.D. *

Assoc. Prof., Department of International Logistics Management, Faculty of Business Administration, Yaşar University, İzmir, Turkey. yucel.ozturkoglu@yasar.edu.tr

* Yaşar Üniversitesi Selçuk Yaşar Kampüsü, Üniversite Cad. No: 37-39 Ağaçlı Yol Bornova, İzmir, Turkey

ABSTRACT

The problem of nurse scheduling is categorized in an Np-Hard complexity as it is inherently composed of many limitations and assumptions. As the number of nurses and the number of days increase, finding the solution of the problem becomes quite difficult. Therefore, this paper propose both an integer-programming model and a Lagrangian relaxation approach for solving nurse-scheduling problem. Numerical results show that while the developed mathematical model works on small-scale problems, Lagrangian relaxation method finds better results for large scale scheduling problem with much smaller duality gap in a reasonable computational time.

Keywords: Nurse, Scheduling, Lagrangian Relaxation, Mathematical Model

1. Introduction

In order for the service provided in hospitals to be of higher quality, the working hours of nurses should be adjusted according to the patient density (Saygili and Ozturkoglu, 2020). As the number of nurses is low, it will increase the workload, decrease the quality of service, and cause many negative results. However, the high number of nurses will significantly increase hospital costs. A large part of the hospital budget consists of salaries paid to nurses. In addition, there is a direct relationship between nurses' working conditions and weekly working hours and their quality of service has led the research to focus on this issue (Moslemi et al. 2017). Today, the working hours of nurses are adjusted with the dual shift system, which is accepted in both private and public hospitals in Turkey and all around the world.

The purpose of scheduling doctors and nurses in hospitals, which is one of the most important of service businesses, is to provide high quality service (Ozbekler and Ozturkoglu, 2020) continuity by allocating hospital employees to shifts and working days with a systematic method. When the studies on the scheduling of healthcare workers are examined, only the appointments of the doctors in general have been studied (Cummings and Shelton, 2002; Robinson and Chen, 2003; Anderson et al. 2015; Bowers et al. 2016; Erhard et al. 2018). However, the studies for nurses have begun much earlier and involve many different subjects and methods.

The concept of nurse scheduling was first introduced in the 1960s. Wolfe and Young (1965) developed first mathematical model, which is aimed to minimize the cost of appointing nurses to different tasks (Wolfe and Young, 1965: 300). Howell (1966) developed the first round model widely used in nursing scheduling. Based on these two studies; different models have been developed for many different purposes in which nurses and hospital requests are handled at the same time or separately. Some of these goals are consecutive holidays (Azaiez and Sharif , 2005; Glass and Knight, 2010), working alternately (Cheang et al. 2003; Burke et al. 2004; Hidri and Labidi, 2016), arranging days on demand (Moz and Pato, 2004; Bard and Purnomo, 2005; Maenhout and Vanhoucke, 2012) and weekend holidays (De Grano et al. 2009; Öztürkoğlu and Çalışkan, 2014; Thongsanit et al. 2015).

For the nursing scheduling problems, classical mathematical models were first applied. However, as the structure of the problems has become more complex, both heuristic (Randhawa and Sitompul, 1993; Beliën, 2007; Ozturkoglu & Bulfin, 2011; Ozturkoglu, 2015; Youssef & Senbel, 2018) and meta-heuristic (Dowsland, & Thompson, 2000; Aickelin, & Dowsland, 2004; Maenhout, & Vanhoucke, 2007; Jafari & Salmasi, 2015; El Adoly et al. 2018) algorithms have been applied recently.

In this study, contrary to the methods used in the studies mentioned above, the nurse-scheduling problem was solved for the first time with a different approach. This problem, which is in the Np-Hard complexity due to the nature of the problem, is applied to be solved for the first time by using Lagrangian relaxation in a reasonable time. Therefore, maximizing the nurse satisfaction and minimizing the related costs of the nurse-scheduling problem are the two important aim of this paper.

This study consists of four main sections. In the second part, Problem Definition, in the third part Methodology, in the fourth part Application of the Model and in the fifth part the results of the application are explained.

2. Problem Definition

As in all jobs, health care providers need to be more careful and vigorous during shifts. Especially nurses' performance is very important for another person's life. The working schedule of nurses in hospitals is prepared weekly or monthly by the authorized person. This process consists of many constraints specific to each hospital, such as legal constraints, personnel policies, nurse expectations and the like. Therefore, as the number of nurses and constraints increases, the scheduling process becomes more difficult.

The main problem of this study is to schedule a set of n numbers of nurse in a specific time interval. It is aimed that the working nurses be assigned to the shifts determined in accordance with the working rules of the hospital in a fair and balanced manner. There are some general assumptions to achieve this aim.

- Service is required 24 hours a day.
- A shift is defined by the start time and duration. Two shifts work day and night.
- The number of nurses is fixed and does not change.
- The minimum nurse demand is strictly met.
- Nurse cannot be assigned to day shift the day after night shift.
- The rest interval during the day is not considered in the model.

In the next section, the mathematical model will be established under different assumptions and constraints. then the established model will be tried to be solved with different approaches.

3. Methodology

Since many scheduling problems are very difficult to solve in real life applications, such problems are called as NP-hard problems. Many different methods are applied to solve such NP-Hard problems. Nurse scheduling problem is one of the most important problem to be solved due to different constraints. Also, the nature of the problem, it is categorized as NP-Hard. In this section, Lagrangian relaxation and traditional mathematical model for the nurse scheduling is discussed.

3.1. Mathematical Model

The indices, parameters and decision variables used in the model are as follows;

Indices

i: 1,….,n nurse index (n: #of nurse)

j:1,……,m day index (m: #of day)

Parameters

aij: cost of i th nurse who works on day j in the day shift

pij: cost of i th nurse who works on night j in the day shift

cij: i th nurse can work on day j in the day shift

bij: i th nurse can work on day j in the night shift

ki: the number of working hour for each nurse

ti: the demand of nurse on day shift

gi: the demand of nurses on night shift

Variables

dij: 1 if nurse i works on day j at day shift

if nurse i does not work on day j at day shift

nij: 1 if nurse i works on day j at night time

if nurse i does not work on day j at night shift

The objective function and constraints of the proposed model are as follows;

Objective Function

Minimize $\sum_{i}^{n}\sum_{j}^{m} c_{ij}d_{ij} + p_{ij}n_{ij}$

Constraints

1) $\sum^{m} a_{ij}d_{ij} + \sum_{j}^{m} p_{ij}n_{ij} \leq k_i$ i=1….n

2) $\sum^{n} d_{ij} = t_j$ j=1,…m

3) $\sum_{i}^{n} n_{ij} = g_j$ j=1,…m

4) dij, nij $\in \{0.1\}$

Objective function is minimizing the total cost of nurses who work in day and night shift. Constraint set (1) ensures that each nurse can work limited hours in both day and night shift. Constraint set (2) ensures that the demand for nurses on day shift must be satisfies and constraint (3) ensures that the demand for nurses on night shift must be satisfies. The formulation of the model is completed by adding the condition 0-1 for all variables.

3.2. Lagrangian Relaxation

In such problems, it is necessary to solve the problem by simplifying the initial hardest problem and obtaining boundaries with different approaches. Lagrangian relaxation is one of the important relaxation techniques that approximates a difficult problem of constrained optimization by a simpler problem (Nocedal and Wright, 2006). A solution to the relaxed problem is an approximate solution to the original problem.

In a branch and bound application, tight bounds lead to fast pruning of the search tree. Therefore, for some Np-Hard problems, we can achieve stricter limits than those obtained by LP relaxation with Lagrangian relaxation. Because of this feature, Lagrangian relaxation provides results closer to the optimal results than other methods.

Consider the following optimization problem where $f: R^n \rightarrow R$ and $S \subseteq R^n$

where $f_R: R^n \rightarrow R$ is such that $f_R(x) \leq f(x)$ for any $x \in S$ and $S \subseteq S_R$.

Minimize $f(x)$ A relaxation of the problem Minimize $f_R(x)$

st $x \in S$ $\longrightarrow$ st $x \in S_R$

As seen in the example, the optimal solution f_R^* of the relaxation is a lower bound of the optimal solution of the initial problem.

The main idea of Lagrangian relaxation is a method of decomposition ($S = S_1 \cup S_2$) the constraints of the problems are separated into two groups. The basis of this method; the constraints of the problem are to separate two groups under the easy (S_1) and hard (S_2) names. Later, the hard constraints are removed and transferred into objective function ($S_R = S_1$). Thus S_R is a set of easy constraints and it will be possible to solve the relaxation problem. Also, the optimal solution of the relaxed problem gives the optimal solution of the initial problem.

The logic of nurse problems is similar to assignment problems. Therefore, when using lagrangian relaxation method, it can be considered as if solving the assignment problem. There are two natural lagrangian relaxations for the generalized assignment problem. The first one is obtained by dualizing constraints, which is used for our proposed model. The general formula of relaxation is given below;

$$v(L_u) = \min\left(\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} + \sum_{i=1}^{n} u_i\left(\sum_{j=1}^{m} a_{ij}x_{ij} - b_i\right)\right) \tag{1}$$

In the next section, nurse scheduling problem will be solved by lagrangian relaxation approach.

3.3. Lagrangian Relaxation Model for Nurse Scheduling

Although there are many studies about nurse scheduling, there are no studies using this Lagrangian relaxation approach. However, this method has been applied in different areas of the health sector. Zhou et al. (2016) developed bi-objective model for the scheduling problem of operating theatres by using relaxation approach. Fathollahi-Fard et al. (2018) has been employed relaxation in the area of home health care considering a policy for overall distance by different type of vehicles.

To the best of our knowledge, this is the first study to developed a Lagrangian relaxation model for solving the nurse scheduling problem. Thus, generalize the same problem with using Lagrangian relaxation approach.

Objective Function

$$\text{Min } -\sum_{i=1}^{n} u_i k_i + \sum_{j=1}^{m}\sum_{i=1}^{n}\left(c_{ij} + u_i a_{ij}\right)\left(d_{ij} + n_{ij}\right) + \sum_{i=1}^{n}\sum_{j=1}^{m}\left(c_{ij} + u_{ij} b_{ij}\right)\left(d_{ij} + n_{ij}\right)$$

Constraints

1) $\sum_{i}^{n} d_{ij} = t_j$ j=1,...m

2) $\sum_{i}^{n} n_{ij} = g_j$ j=1,...m

Proposed relaxation model has some differences from the previous mathematical model. In relaxation model, objective function is changed and first constraint is removed. First constraint is added in the objective function because of Lagrangian relaxations. In addition, because of the integrality, 4th constraint is removed in the new model. From the formulation of the problems, all data are nonnegative integers.

In the next section, this developed model will be solved with real data.

4. Application of the Models

In order to test the developed models, nurse scheduling will be done in Eye clinic of a private hospital in İzmir. Nurse scheduling will be done with the assumptions and constraints given in the previous section remaining the same. There are four nurses in the department where scheduling will be performed. Scheduling will be tested for a period of five days. Objective function is minimizing the total cost of nurses who work in day and night shift.

	Day Shift					**Night Shift**				
Days	1	2	3	4	5	1	2	3	4	5
Number of nurses required	2	3	1	2	1	2	1	1	2	3

Table 1. Information about Day and Night Shifts

The data for the application of the developed model on the sample is shown as follows.

Indices

i: 1,....,4 nurse index

j:1,......,5 day index

The pure 0-1 mathematical programming model is written for the sample problem. The proposed mathematical model is coded using AMPL and solved by CPLEX 9.1 on a computer with a Pentium IV 2.8-GHz processor and 1 GB of RAM. When the solution of the model is examined, it is seen that the value of the objective function is 84. Table 2 and Table 3 show the mathematical model results of the day and night shift.

(Nurse, Day)	Value	Reduced Cost	(Nurse, Day)	Value	Reduced Cost
(1,1)	0.00	5.00	(3,1)	1.00	4.00
(1,2)	0.00	3.00	(3,2)	1.00	2.00
(1,3)	1.00	1.00	(3,3)	0.00	6.00
(1,4)	1.00	6.00	(3,4)	0.00	7.00
(1,5)	1.00	2.00	(3,5)	0.00	8.00
(2,1)	1.00	3.00	(4,1)	0.00	5.00
(2,2)	1.00	4.00	(4,2)	1.00	3.00
(2,3)	0.00	6.00	(4,3)	0.00	7.00
(2,4)	0.00	9.00	(4,4)	1.00	8.00
(2,5)	0.00	6.00	(4,5)	0.00	4.00

Table 2. Mathematical Model Results for Day Shift

(Nurse, Day)	Value	Reduced Cost	(Nurse, Day)	Value	Reduced Cost
(1,1)	0.00	7.50	(3,1)	1.00	6.00
(1,2)	0.00	1.50	(3,2)	1.00	3.00
(1,3)	1.00	1.50	(3,3)	0.00	7.50
(1,4)	1.00	7.50	(3,4)	1.00	10.50
(1,5)	1.00	3.00	(3,5)	0.00	12.00
(2,1)	1.00	4.50	(4,1)	0.00	7.50
(2,2)	0.00	6.00	(4,2)	0.00	4.50
(2,3)	0.00	7.50	(4,3)	0.00	10.00
(2,4)	0.00	13.50	(4,4)	0.00	12.00
(2,5)	1.00	9.00	(4,5)	1.00	6.00

Table 3. Mathematical Model Results for Night Shift

The same problem was solved by the Lagrangian relaxations approach at this time. In the relaxation, parameter *u* value is given randomly. The approach is coded using AMPL and solved by CPLEX 9.1 on a computer with a Pentium IV 2.8-GHz processor and 1 GB of RAM. The objective function of this model is 84 and model gives integer solution without integer constraint.

v(P) $\geq v(L_u)$

Table 4 and Table 5 show the relaxation approach results of the day and night shift.

(Nurse, Day)	Value	Reduced Cost	(Nurse, Day)	Value	Reduced Cost
(1,1)	0.00	2.00	(3,1)	1.00	0.00
(1,2)	0.00	2.00	(3,2)	1.00	0.00
(1,3)	1.00	0.00	(3,3)	0.00	10.00
(1,4)	0.00	0.00	(3,4)	0.00	2.00
(1,5)	1.00	0.00	(3,5)	0.00	12.00
(2,1)	0.00	7.60	(4,1)	0.00	6.00
(2,2)	0.00	15.20	(4,2)	0.00	4.00
(2,3)	0.00	24.4	(4,3)	0.00	15.60
(2,4)	0.00	15.60	(4,4)	0.00	7.20
(2,5)	0.00	20.79	(4,5)	0.00	8.40

Table 4. Relaxation Model Results for Day Shift

When the two objective functions are compared, it is seen that the results are equal to each other.

(Nurse, Day)	Value	Reduced Cost	(Nurse, Day)	Value	Reduced Cost
(1,1)	0.00	2.00	(3,1)	1.00	0.00
(1,2)	0.00	2.00	(3,2)	1.00	0.00
(1,3)	1.00	0.00	(3,3)	0.00	10.00
(1,4)	1.00	0.00	(3,4)	0.00	2.00
(1,5)	1.00	0.00	(3,5)	0.00	12.00
(2,1)	0.00	7.60	(4,1)	0.00	6.00
(2,2)	0.00	15.20	(4,2)	0.00	4.00
(2,3)	0.00	24.40	(4,3)	0.00	15.60
(2,4)	0.00	15.60	(4,4)	0.00	7.20
(2,5)	0.00	20.79	(4,5)	0.00	8.40

Table 5. Relaxation Model Results for Night Shift

According to results, Lagrangian relaxations approach works very well with a reasonable time. If we increase the nurse number, mathematical model may not be solved in a reasonable time. On the other hand, relaxations approach can be used instead of mathematical model.

5. Conclusion

The nurse scheduling in hospitals is prepared weekly or monthly by the authorized person. This process consists of many constraints specific to each hospital, such as legal constraints, personnel policies, nurse expectations and the like. Therefore, as the number of nurses and constraints increases, the scheduling process becomes more difficult. This paper proposed both mathematical model and Lagrangian relaxation model to solve nurse scheduling problem. Proposed solution methods are tested on real-world example in Izmir. According to results, the mathematical model is used for small-scale problems, while Lagrangian relaxation approach is best for large-scale problems in a reasonable time. The main limitation of this study is the number of the nurse and the number of the day. For future studies, proposed model can be solved for slightly larger samples. In addition, assumptions can be changed to be specific to each hospital. Therefore, there are a wide range of directions for possible future research in nurse scheduling problem. Especially for large-scale problems, it is necessary to try to solve this model by using other relaxation approaches.

References

Aickelin, U., & Dowsland, K. A. (2004). An indirect genetic algorithm for a nurse-scheduling problem. *Computers & Operations Research*, 31(5), 761-778.

Anderson, K., Zheng, B., Yoon, S. W., & Khasawneh, M. T. (2015). An analysis of overlapping appointment scheduling model in an outpatient clinic. *Operations Research for Health Care*, 4, 5-14.

Azaiez, M. N., & Al Sharif, S. S. (2005). A 0-1 goal programming model for nurse scheduling. *Computers & Operations Research*, 32, 491-507.

Bard, J., & Purnomo, H. (2005). Short-term nurse scheduling in response to daily fluctuations in supply and demand. *Health Care Management Science*, 8, 315-324.

Beliën, J. (2007). Exact and heuristic methodologies for scheduling in hospitals: problems, formulations and algorithms (Doctoral dissertation, Springer-Verlag).

Bowers, M. R., Noon, C. E., Wu, W., & Bass, J. K. (2016). Neonatal physician scheduling at the University of Tennessee Medical Center. *Interfaces*, 46(2), 168-182.

Burke E. K., De Causmaecker P., Vanden Berghe G., &Van Landeghem H. (2004). The state of the art of nurse rostering. *Journal of Scheduling* 7, 441–499.

Cheang B., Li H., Lim A., & Rodrigues B. (2003). Nurse rostering problems—a bibliographic survey. *European Journal of Operations Research*, 151, 447–460.

Cummings Jr, D. D., & Shelton, R. H. (2002). U.S. Patent No. 6,345,260. Washington, DC: U.S. Patent and Trademark Office.

De Grano, M. L., Medeiros, D. J., & Eitel, D. (2009). Accommodating individual preferences in nurse scheduling via auctions and optimization. *Health Care Management Science*, 12(3), 228.

Dowsland, K. A., & Thompson, J. M. (2000). Solving a nurse scheduling problem with knapsacks, networks and tabu search. *Journal of the Operational Research Society*, 51(7), 825-833.

El Adoly, A. A., Gheith, M., & Fors, M. N. (2018). A new formulation and solution for the nurse scheduling problem: A case study in Egypt. *Alexandria Engineering Journal*, 57(4), 2289-2298.

Erhard, M., Schoenfelder, J., Fügener, A., & Brunner, J. O. (2018). State of the art in physician scheduling. *European Journal of Operational Research*, 265(1), 1-18.

Fathollahi-Fard, A. M., Hajiaghaei-Keshteli, M., & Tavakkoli-Moghaddam, R. (2018). A Lagrangian relaxation-based algorithm to solve a Home Health Care routing problem. *International Journal of Engineering*, 31(10), 1734-1740.

Glass, C. A., & Knight, R. A. (2010). The nurse rostering problem: A critical appraisal of the problem structure. *European Journal of Operational Research*, 202, 379-389.

Hidri L., & Labidi M. (2016). Optimal physicians schedule in an Intensive Care Unit. IOP Conf. Series: *Materials Science and Engineering* 131, 1-8.

Howell, J. P. (1966). Cyclical scheduling of nursing personnel. *Hospitals*, 40(2), 77-85.

Jafari, H., & Salmasi, N. (2015). Maximizing the nurses' preferences in nurse scheduling problem: mathematical modeling and a meta-heuristic algorithm. *Journal of Industrial Engineering International*, 11(3), 439-458.

Maenhout, B., & Vanhoucke, M. (2007). An electromagnetic meta-heuristic for the nurse scheduling problem. *Journal of Heuristics*, 13(4), 359-385.

Maenhout, B., & Vanhoucke, M. (2012). An integrated nurse staffing and scheduling analysis for longer-term nursing staff allocation problems. *Omega*, 41(2), 485-499.

Moslemi, S., Sabegh, M. H. Z., Mirzazadeh, A., Ozturkoglu, Y., & Maass, E. (2017). A multi-objective model for multi-production and multi-echelon closed-loop pharmaceutical supply chain considering quality concepts: NSGAII approach. *International Journal of System Assurance Engineering and Management*, 8(2), 1717-1733.

Moz, M., & Pato, M. V. (2004). Solving the problem of re-rostering nurse schedules with hard constraints: New multicommodity flow models. *Annals of Operations Research*, 128(1-4), 179-197.

Nocedal, J., & Wright, S. J. (2006). Numerical Optimization, 2nd Edition, New York: Springer.

Ozbekler, T. M., & Ozturkoglu, Y. (2020). Analyzing the importance of sustainability-oriented service quality in competition environment. *Business Strategy and the Environment*, 29(3), 1504-1516.

Ozturkoglu, Y. (2015). An efficient time algorithm for makespan objectives. *An International Journal of Optimization and Control: Theories & Applications (IJOCTA),* 5(2), 75-80.

Ozturkoglu, Y., & Bulfin, R. L. (2011). A unique integer mathematical model for scheduling deteriorating jobs with rate-modifying activities on a single machine. *The International Journal of Advanced Manufacturing Technology*, 57(5-8), 753-762.

Öztürkoğlu, Y., & Çalışkan, F. (2014). Hemşire çizelgelenmesinde esnek vardiya planlanması ve hastane uygulaması. *Dokuz Eylül Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, 16, 115-133.

Randhawa, S. U., & Sitompul, D. (1993). A heuristic-based computerized nurse scheduling system. *Computers & Operations Research*, 20(8), 837-844.

Robinson, L. W., & Chen, R. R. (2003). Scheduling doctors' appointments: optimal and empirically-based heuristic policies. *IIE Transactions*, 35(3), 295-307.

Saygili, E., & Ozturkoglu, Y. (2020). Patients' rights and professional conduct issues in hospitals' codes of ethics. *International Journal of Human Rights in Healthcare*, 13(3), 201-208.

Thongsanit, K., Kantangkul, K., & Nithimethirot, T. (2015). Nurse's shift balancing in nurse scheduling problem. *Silpakorn U Science & Tech J*, 10, 43-48, 2015.

Wolfe, H., & Young, J. P. (1965). Staffing the nursing unit: Part I. controlled variable staffing. *Nursing Research*, 14(3), 236-242.

Youssef, A., & Senbel, S. (2018, January). A bi-level heuristic solution for the nurse-scheduling problem based on shift swapping. In 2018 IEEE 8th Annual Computing and Communication Workshop and Conference (CCWC) (pp. 72-78), IEEE.

Zhou, B. H., Yin, M., & Lu, Z. Q. (2016). An improved Lagrangian relaxation heuristic for the scheduling problem of operating theatres. *Computers & Industrial Engineering*, 101, 490-503.