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Research Article

Hyperbolic Type Traveling Wave Solution of Vakhnenko-Parkes Equation

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Abstract

In this article, structure of $(1/G')$-expansion method is applied. Another name of the reduced Ostrovsky equation, Vakhnenko-Parkes (V-P) equation is taken into consideration and exact solutions have been constructed of the (V-P) equation using $(1/G')$-expansion method. This method is an easier and efficient method for finding analytic solutions of nPDEs. The method appears to be easier and faster for symbolic computation.

Keywords: $(1/G')$-expansion method, the Vakhnenko-Parkes equation, exact solution, traveling wave solution.

Vakhnenko-Parkes Denkleminin Hiperbolik Tipte Yürüyen Dalga Çözümü

Öz

Bu makalede, $(1/G')$-açılım metodunun yapısı uygulanmıştır. İndirgenmiş Ostrovsky denkleminin bir diğer adı olan Vakhnenko-Parkes (V-P) denklemi dikkate alınmış ve (V-P) denkleminin $(1/G')$-açılım metodunu kullanılarak tam çözümleri inşa edilmiştir. Bu yöntem lineer olmayan kısmi diferansiyel denklemlerin analitik çözümlerini bulmak için daha kolay ve etkili bir metottur. Metot sembolik hesaplama için daha kolay ve daha hızlı görünüyor.

Anahtar Kelimeler: $(1/G')$-açılım metodu, Vakhnenko-Parkes denklemi, tam çözüm, yürüyen dalga çözümü.

1. Introduction

nPDEs are studied in many areas such as in fluids science, engineering, and applied sciences. Many studies on analytical and numerical solutions have been done in literature for long years. A variety of methods are used for these studies. Some of these methods are Hirota bilinear method Manafian, (2018), The tanh–coth method Wazwaz (2007), (G'/G)-expansion method Yokuş and Kaya (2015), Yokus and Tuz (2017), Durur (2019), Sumudu transform method Yavuz and Özdemır (2018), extended sinh-Gordon equation expansion method Baskonus et al. (2018), Cattani et al. (2018), Sub equation method Durur et al. (2019), the Clarkson–Kruskal (CK) direct method Su-Ping and Li-Xin (2007), $(1/G')$-expansion method Yokuş and Durur (2019), Yokuş and Kaya (2015), Durur and Yokuş (2019), the modified Kudryashov method Kumar et al. (2018), Adomian Decomposition methods

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Kaya and Yokus (2002), Kaya and Yokus (2005), Yavuz and Özdemir (2018), first integral method Darvishic et al. (2016), collocation method Aziz and Šarler (2010), new sub equation method Kurt et al. (2019), improved Bernoulli sub-equation function method Baskonus and Bulut (2016), Dusunceli (2019), residual power series method Durur et al. (2019), the modified exp-expansion function method Yokus et al. (2018), Difference scheme method Faraj and Modanli (2017), newly extended direct algebraic technique Gao et al. (2020) and so on.

Vakhnenko-Parkes confirmed the reduced Ostrovsky equation may be converted to the new integrable equation (Vakhnenko and Parkes,1998).

Consider the (V-P) equation Abazari (2010),

$$uu_{xxt} - u_x u_{xt} + u^2 u_t = 0, \tag{1}$$

in the form.

Many scientists have been studied with (V-P) equation. Some of these are as follows: solutions of the (V-P) equation are obtained using the inverse scattering method Vakhnenko and Parkes (2012), with the help of exp-function and the exp-$(-\phi(\xi))$-expansion method, the exact solutions of the (V-P) equation are obtained Roshid et al. (2014), two solitary wave solutions of (V-P) equation are attained using the ansatz method Majid et al. (2012), different type solutions of (V-P) equation are attained Ye et al. (2012), analytic solutions to the (V-P) equation using the complex method Gu et al. (2017), exact solutions of the (V-P) equation are obtained using the improved (G'/G) expansion method Liu and He (2013), different type solutions of the (V-P) equation are obtained with the aim of the Bernoulli sub-equation function method (Baskonus et al.,2015).

In this study, we are interested in the (V-P) equation. We have been obtained analytic solutions for the (V-P) equation using $(1/G')$-expansion method.

2. $(1/G')$-Expansion Method

Consider general form of NLPDEs

$$T\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \dots\right) = 0. \tag{2}$$

Let

$$u = u(x,t) = U(\xi), \;\; \xi = kx + wt, w \neq 0,$$

where w, k are constants and w is the velocity of the wave. We may be converted into following nODE for $U(\xi)$:

$$\mathrm{L}(U, U', U'', U''', \dots) = 0. \tag{3}$$

The solution of Eq. (3) is assumed to have the form

$$U(\xi) = a_0 + \sum_{i=1}^{n} a_i \left(\frac{1}{G\prime}\right)^i, \tag{4}$$

where a_i are constants, n is a positive integer which is the equilibrium term in Eq. (3) and $G = G(\xi)$ provides the following second-order lODE

$$G'' + \lambda G' + \mu = 0, \tag{5}$$

where λ and μ are constants to be determined after

$$\frac{1}{G'[\xi]} = \frac{1}{-\frac{\mu}{\lambda} + A\cos h[\xi\lambda] - A\sin h[\xi\lambda]}, \tag{6}$$

where A is an integral constant. If the desired derivatives of the Eq. (4) are calculated and replacing in the Eq. (3), a polynomial with the argument $(1/G')$ is attained. An algebraic equation system is created by equalizing the coefficients of this polynomial to zero. These equations are solved using the package program and put into place in the default Eq. (3) solution function. Thus, the solutions of Eq. (1) are attained.

3. Solutions of the (V-P) equation

We consider Eq. (1) and using transformation

$u = u(x,t) = U(\xi), \quad \xi = kx + wt, \; w \neq 0,$

where w, k are constants

$$k^2 wUU''' - k^2 wU'U'' + wU^2U' = 0. \quad (7)$$

If the Eq. (7) is integrated according to ξ, we can write the following equation

$$3k^2UU'' - 3k^2(U')^2 + U^3 = 0. \quad (8)$$

Here, the integral constant is zero. According to the homogeneous balancing principle, the equilibrium term of Eq. (8) is $n = 2$ and the following situation is obtained in Eq. (4),

$$U(\xi) = a_0 + a_1\left(\frac{1}{G'[\xi]}\right) + a_2\left(\frac{1}{G'[\xi]}\right)^2, a_2 \neq 0. \quad (9)$$

Replacing Eq. (9) into Eq. (8) and the coefficients of Eq. (1) are equal to zero, we may establish the following algebraic equation systems

Const: $a_0^3 = 0,$

$\frac{1}{G'[\xi]}: 3k^2\lambda^2 a_0 a_1 + 3a_0^2 a_1 = 0,$

$\frac{1}{G'[\xi]^2}: 9k^2\lambda a_0 a_1 + 3a_1^2 a_0 + 12k^2\lambda^2 a_0 a_2 + 3a_0^2 a_2 = 0,$

$\frac{1}{G'[\xi]^3}: 6k^2\mu^2 a_0 a_1 + 3k^2\lambda\mu a_1^2 + a_1^3 + 30k^2\lambda\mu a_0 a_2 + 3k^2\lambda^2 a_1 a_2 + 6a_0 a_1 a_2 = 0,$

$\frac{1}{G'[\xi]^4}: 3k^2\mu^2 a_1^2 + 18k^2\mu^2 a_0 a_2 + 15k^2\lambda\mu a_1 a_2 + 3a_1^2 a_2 + 3a_0 a_2^2 = 0,$

$\frac{1}{G'[\xi]^5}: 12k^2\mu^2 a_1 a_2 + 6k^2\lambda\mu a_2^2 + 3a_1 a_2^2 = 0,$

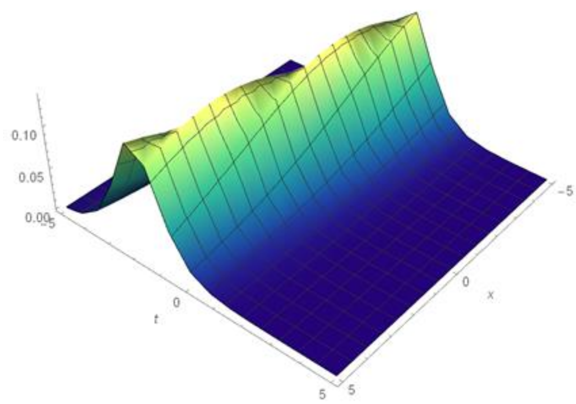
$\frac{1}{G'[\xi]^6}: 6k^2\mu^2 a_2^2 + a_2^3 = 0. \quad (10)$

Case1.

$a_0 = 0, a_1 = -6k^2\lambda\mu, a_2 = -6k^2\mu^2, \xi = kx + wt, \quad (11)$

replacing values Eq. (11) into Eq. (9) and obtain the following hyperbolic wave solutions for Eq. (1):

$$u(x,t) = -\frac{6k^2\mu^2}{\left(-\frac{\mu}{\lambda} + A\mathrm{Cosh}[\lambda\,\xi] - A\mathrm{Sinh}[\lambda\,\xi]\right)^2} - \frac{6k^2\lambda\mu}{-\frac{\mu}{\lambda} + A\mathrm{Cosh}[\lambda\,\xi] - A\mathrm{Sinh}[\lambda\,\xi]}. \quad (12)$$



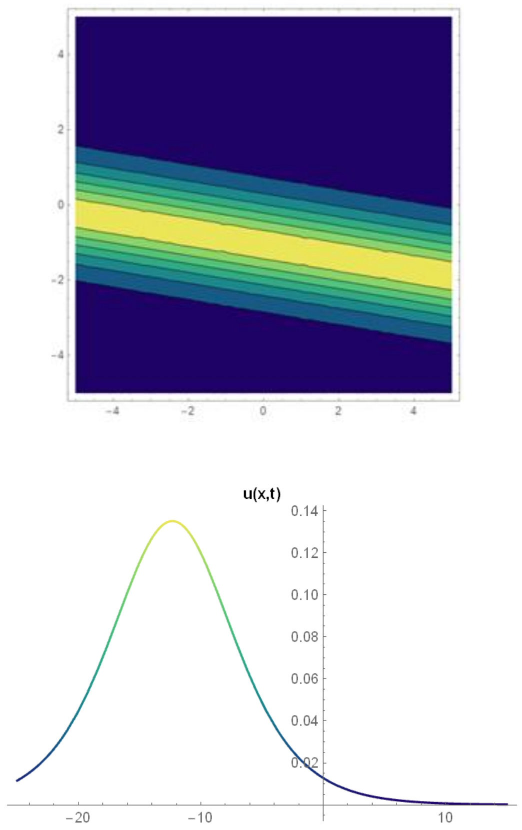


Figure 1. 3D, contour and 2D graphs of $u(x,t)$ respectively for $\mu=-2,\ \lambda=3,\ w=0.6,\ k=0.1,\ A=0.1$.

4. Conclusion

In this letter, the new hyperbolic traveling wave solution of (V-P) equation has been obtained using $(1/G')$-expansion method. The biggest advantage of the $(1/G')$-expansion method is that it can be easily applied to many nonlinear partial differential equations. In addition, the systems of equations obtained by many expansion methods are simpler. It is a reliable and powerful method in obtaining traveling wave solutions. The disadvantage of this method is the production of a uniform wave solution. The solutions obtained are hyperbolic traveling wave solutions and are in Eq. (6) format.

Moreover, the computer package program has been used for computations and graphics in this study.

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